

Analytical Performance Evaluation of an Optical Heterodyne CPFSK System Impaired by Polarization Mode Dispersion in a Single Mode Fiber

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Abstract—A theoretical analysis is developed to evaluate the bit error rate (BER) performance degradation of an optical heterodyne CPFSK system caused by signal phase distortion due to polarization mode dispersion (PMD) in a single mode fiber. The average BER performance results are evaluated at a bit rate of 10 Gb/s considering Maxwellian distribution for the differential group delay (DGD). The results show that the performance of optical heterodyne CPFSK direct detection system suffers power penalty of 0.20 dB, 0.70 dB, 2.40 dB and 7.25 dB corresponding to mean DGD of 20 ps, 30 ps, 40 ps and 60 ps respectively at modulation index 0.75 for a BER of 10^{-9} operating at 10 Gb/s. Furthermore, at increased values of the mean DGD there occur BER floors above 10^{-9} , which can not be lowered by further increasing the signal power. It is noticed that BER floors occur at about 2×10^{-7} , 10^{-5} and 2.5×10^{-4} corresponding to mean DGD of 70 ps, 80 ps and 90 ps respectively at a modulation index of 1.50. The effect of PMD is found to be more detrimental at higher modulation index

I. INTRODUCTION

Optical continuous-phase FSK (CPFSK) with low modulation index ($h \leq 1$) is an attractive modulation scheme for future multi-channel optical system. However, at high bit rate operation of such systems in conventional single mode fiber, the major limitation is the group velocity dispersion (GVD), unless a dispersion compensation scheme is used [1]. Further, a single mode fiber is actually a two-mode fiber because it can support two orthogonally polarized eigenmodes. Due to imperfections in the fiber manufacture and deployment, the propagation constants of the two eigenmodes are in general different and mode coupling occurs as light propagates through a fiber. The different propagation speeds result in PMD and maximum dispersion happens when two modes are equally excited. Consequently, an incident linearly polarized wave may be subjected to random phase fluctuation which causes spectral broadening of the transmitted signal that leads to bit pattern corruption and higher bandwidth requirement, and eventually contributes to BER deterioration, performance variation or system fading even at moderate bit rates [2]-[9].

PMD constitutes one of the main limiting factors for reliable optical fiber system transmission performance at bit rate 10 Gb/s or higher. The effect of PMD has been the subject of considerable research interests during the past few years and the developed ideas are briefly described in a series of publications [3]-[9]. The simulation results on the effects of PMD on an amplified IM-DD system is reported in ref. [3] as a function of the DGD. The effect of PMD on the conditional bit error rate performance of heterodyne FSK system is presented [6]. Recently, the performance of optical DPSK system with direct detection receiver is reported in presence of PMD [7].

In this paper, we provide an analytical approach to evaluate the BER performance limitations of an optical heterodyne CPFSK with delay-demodulation system impaired by PMD. We determine the average BER and power penalty for different mean values of DGD, considering Maxwellian probability density function for the DGD, at the receiver output in the presence of receiver noise. The bit error rate performance and power penalty suffered by the system due to PMD at a BER of 10^{-9} are evaluated at a bit rate of 10 Gb/s.

II. RECEIVER MODEL

The block diagram of a CPFSK delay demodulation receiver is shown in Fig.1. The received optical signal is combined with the optical signal output from the local oscillator and the two signals are detected by photo-detector. The output of the photodetector which is an intermediate frequency (IF) signal is amplified by the receiver pre-amplifier and then filtered by a gaussian filter with center frequency set to the IF. The bandwidth of the IF is kept twice the bit rate for optimum demodulation. The output of the filter is then demodulated using a delay line discriminator. The output of the discriminator is then filtered by a low-pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero than a

binary '1' is detected, otherwise a binary '0' is detected.

III. THEORITICAL ANALYSIS

The complex electric field at the output of the continuous phase FSK (CPFSK) transmitter and input to the fiber is represented as:

$$E_1(t) = \sqrt{2P_T} \exp[j2\pi f_c t + j\phi_s(t)][c_1 \cdot e_1] \quad (1)$$

$$E_2(t) = \sqrt{2P_T} \exp[j2\pi f_c t + j\phi_s(t)][c_2 \cdot e_2] \quad (2)$$

where, f_c is the carrier frequency, P_T is the transmitted optical power and the CPFSK modulating phase $\phi_s(t)$ is given by

$$\phi_s(t) = 2\pi \Delta f \int_{-\infty}^t I(t) dt + \phi_n(t), \text{ and}$$

$$I(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

with $a_k = \pm 1$ is the k th bit of random NRZ bit pattern, Δf , the peak frequency deviation, $p(t)$, the elementary pulse shape of duration T seconds and $\phi_n(t)$, the phase noise of the transmitting laser. Here, c_1, c_2 represent unit vectors and e_1, e_2 represent the two principal states of polarization (PSPs) respectively.

The electric field output of the fiber is then given by

$$E_{01}(t) = \sqrt{2P_T} \exp[j2\pi f_c t + j\phi_s(t)][c_1 \cdot e_1] \otimes h_1(t) \quad (3)$$

$$E_{02}(t) = \sqrt{2P_T} \exp[j2\pi f_c t + j\phi_s(t)][c_2 \cdot e_2] \otimes h_2(t) \quad (4)$$

where $\otimes$ denotes convolution, $h_1(t)$ and $h_2(t)$ are the inverse Fourier transform of the low pass equivalent transfer function of a non-dispersion shifted lossless fiber $H_1(f)$ and $H_2(f)$ respectively, which include the effect of PMD and group velocity dispersion and are given by,

$$H_1(f) = \sqrt{\alpha} \exp[j2\pi f(-\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \quad (5)$$

$$H_2(f) = \sqrt{1-\alpha} \exp[j2\pi f(\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \quad (6)$$

where α is the PMD power splitting ratio, $\Delta\tau$ represents the DGD between the two PSPs and γ is chromatic dispersion index. Here, we assume that there is a negligible amount of polarization dependent loss.

Now, assuming linear phase approximation, the output electric fields for two polarization states can be given as,

$$E_{01}(t) = \sqrt{2P_s} \exp[j2\pi f_c t + j\phi_{01}(t)][c_1 \cdot e_1] \quad (7)$$

$$E_{02}(t) = \sqrt{2P_s} \exp[j2\pi f_c t + j\phi_{02}(t)][c_2 \cdot e_2] \quad (8)$$

where,

$$\phi_{01}(t) = 2\pi \Delta f \int_{-\infty}^t \sum_k a_k \text{Re}[p(t - kT) \otimes h_1(t)] dt \quad (8a)$$

$$\phi_{02}(t) = 2\pi \Delta f \int_{-\infty}^t \sum_k a_k \text{Re}[p(t - kT) \otimes h_2(t)] dt \quad (8b)$$

The signal at the output of the IF filter is given by,

$$i_0(t) = R_d \sqrt{P_s P_{Lo}} \{ \exp[j2\pi f_{IF} t + j\phi'_{01}(t)] + \exp[j2\pi f_{IF} t + j\phi'_{02}(t)] \} + i_n(t) \quad (9)$$

where $i_n(t)$ represents the total noise current consisting of shot noise and receiver thermal noise, R_d is the responsivity of the photodetector, P_s is the output power at the fiber end, P_{Lo} is the local oscillator power, f_{IF} is the IF frequency. The phases $\phi'_{01}(t)$ and $\phi'_{02}(t)$ are given by

$$\phi'_{01}(t) = 2\pi \Delta f \int_{-\infty}^t \sum_k a_k g_1(t - kT) dt \quad (10)$$

$$\phi'_{02}(t) = 2\pi \Delta f \int_{-\infty}^t \sum_k a_k g_2(t - kT) dt \quad (11)$$

$$\text{where, } g_1(t) = \text{Re}[p(t) \otimes h_1(t) \otimes h_{IF}(t)] \quad (11a)$$

$$g_2(t) = \text{Re}[p(t) \otimes h_2(t) \otimes h_{IF}(t)] \quad (11b)$$

where $h_{IF}(t)$ is the impulse response of the IF filter. The IF signal current is then given by,

$$i_0(t) = 2R_d \sqrt{P_s P_{Lo}} \cos\phi_0^*(t) \cos[2\pi f_{IF} t + \phi_0'(t)] + i_n(t) \quad (12)$$

$$\text{where, } \phi_0'(t) = \frac{\phi'_{01} + \phi'_{02}}{2} \quad \text{and} \quad \phi_0^*(t) = \frac{\phi'_{01} - \phi'_{02}}{2}$$

The output of the IF filter can be expressed as,

$$v_0(t) = 2R_d \sqrt{P_s P_{Lo}} a(t) \cos[2\pi f_{IF} t + \phi_0'(t)] + n(t) \quad (13)$$

where, $a(t) = \cos\phi_0^*(t)$, which is the amplitude fluctuations due to PMD and GVD, $n(t)$ is the receiver shot noise with variance $\sigma_n^2 = 2e(P_s + P_{Lo})B_{IF}$, B_{IF} being the IF filter bandwidth.

Following the delay demodulation (delay time τ) and low pass filtering (wide enough to pass the signal undistorted), the low pass filter (LPF) output is given by,

$$v_{out}(t) = A(t) \cos[\Delta\phi_0(t, \tau)] \quad (14)$$

where $A(t) = 2R_d \sqrt{P_s P_{Lo}} \cos\phi_0^*(t)$; $A(t)$ is the amplitude and the differential output phase of $v_{out}(t)$ of the LPF can be expressed as,

$$\Delta\phi_0(t, \tau) = 2\pi f_{IF} \tau + \Delta\phi_0'(t, \tau) + \Delta\theta_n(t, \tau)$$

$$\Delta\phi_0(t, \tau) = 2\pi \Delta f \left[\int_{t-\tau}^t a_0 g_0(t - kT) dt + \int_{t-\tau}^t \sum_{k \neq 0} a_k g_0(t - kT) dt \right] \quad (15)$$

which can be rewritten as,

$$\Delta\phi_0(t, \tau) = 2\pi \tau \Delta f q(t) + 2\pi \tau \Delta f \sum_{k \neq 0} a_k q(t - kT) \quad (16a)$$

$$= \bar{\Delta}\phi_0(t, \tau) + \xi \quad (16b)$$

where, $q(t) = \frac{1}{\tau} \int_{t-\tau}^t g_0(t) dt$ and $g_0(t) = \frac{1}{2}[g_1(t) + g_2(t)]$

In delay demodulation receiver the data decisions are based on the polarity of $v_{out}(t)$. Assuming a 'mark' is transmitted (say $a_0 = 1$) and under ideal CPFSK demodulation condition, $2\pi f_{IF}\tau = (2n+1)\frac{\pi}{2}$; n is an integer and $2\pi \Delta f\tau = \frac{\pi}{2}$ for NRZ data.

Now, applying the above conditions, the phase of $v_{out}(t)$ at the sampling instant t_0 can be written as,

$$\Delta\phi(t_0, \tau) = 2\pi f_{IF}\tau + \frac{\pi}{2} + \frac{\pi}{2} q(t_0) + \frac{\pi}{2} \sum_{k \neq 0} q(t_0 - kT) + \Delta\theta_n(t_0, \tau) \quad (17)$$

Note that the first two terms in (17) provide the expected phase change during the demodulation interval τ corresponding to the ideal situation, the next three terms, in fact, represent the undesired contribution to the phase due to PMD and GVD and the last term, $\Delta\theta_n(t_0, \tau)$ represents the phase distortion due to receiver noise. Denoting the phase distortion of the signal only due to PMD and GVD as

$$\eta = -\frac{\pi}{2} + \frac{\pi}{2} q(t_0) + \frac{\pi}{2} \sum_{k \neq 0} q(t_0 - kT) = \bar{\Delta} \alpha_0(t, \tau) + \xi \quad (18)$$

where, $\bar{\Delta} \alpha_0(t, \tau)$ is the mean output phase error and ξ is random output phase fluctuation due to ISI for random bit pattern caused by PMD and GVD.

We define the IF SNR as $\rho = \frac{V_0^2}{2\sigma_n^2}$. For a given IF SNR, the conditional bit error probability, $P(e|\xi)$ conditioned on a given value of ξ and differential group delay $\Delta\tau$, can be obtained following Ref. [10].

The probability density function (pdf) of ξ , $P_\xi(\xi)$ can be obtained by inverting the characteristic function of ξ . The characteristic function of random output phase, ξ can be expressed as,

$$F_\xi(j\xi) = \prod_{i=1}^{\infty} \cos[\xi q_i(t)] = \sum_{i=1}^{\infty} \frac{(j\xi)^{2i}}{(2i)!} M_{2i} \quad (19)$$

Where $q_i(t) = |q(t - iT)|$, M_{2i} are the even order moments of the characteristic function of random output phase ξ . Moments M_{2i} can be evaluated by using the following recursive relations,

$$M_{2r} = Y_{2r}(N) \quad (20)$$

$$Y_{2r}(i) = \sum_{j=0}^r {}^{2r}C_{2j} Y_{2j}(i-1) q_i^{2r-2j} \quad (21)$$

Where ${}^{2r}C_{2j}$ is the binominal coefficient and N is the actual number of terms

The pdf of the random output phase, ξ can be written as,

$$P_\xi(\xi) = F^{-1}[F_\xi(j\xi)] \quad (22)$$

So for a given value of instantaneous differential group delay (DGD), $\Delta\tau$ the conditional BER can be expressed as,

So for a given value of differential group delay (DGD), $\Delta\tau$ the conditional BER can be expressed as,

$$BER(\Delta\tau) = \int_{-\infty}^{\infty} P(e|\xi) P_\xi(\xi) d\xi \quad (23)$$

where $P_\xi(\xi)$ is the pdf of ξ which can be evaluated following Ref.[5]. The above integration can be carried out by Gauss-quadrature rule.

The statistical properties of PMD have been experimentally and theoretically studied and found that the evolution of DGD at particular frequency over time yields a Maxwellian probability density function given by,

$$P_{\Delta\tau}(\Delta\tau) = \frac{32\Delta\tau^2}{\pi^2 \langle \Delta\tau \rangle^3} \exp\left[-\frac{4\Delta\tau^2}{\pi \langle \Delta\tau \rangle^2}\right], \quad \Delta\tau > 0 \quad (24)$$

where, $\langle \Delta\tau \rangle$ is the quadratic mean of instantaneous DGD, $\Delta\tau$. Thus, the average error probability can be evaluated as,

$$BER = \int_{-\infty}^{\infty} BER(\Delta\tau) P_{\Delta\tau}(\Delta\tau) d(\Delta\tau) \quad (25)$$

IV. RESULTS AND DISCUSSION

Following the analytical approach outlined above, we evaluated the BER performance result of an optical heterodyne CPFSK transmission system with delay demodulation for several values mean DGD at a bit rate of 10 Gb/s. The plots of average BER versus received power, P_s are shown in Fig. 2 and Fig.3. at modulation index of 0.75 and 1.50 respectively at a bit rate 10 Gb/s for different values of mean DGD. The results show that the BER performance is more affected at higher values of mean DGD and at higher modulation index

The penalty due to PMD suffered by the heterodyne CPFSK system at BER 10^{-9} is plotted in Fig.2 as a function of DGD normalized by bit duration. It is observed from Fig.1 that the penalty at BER = 10^{-9} is increasing with increasing values of the mean DGD. The amount of penalty is found to be approximately 0.75 dB, 2.00 dB, 3.50 dB and

5.70 dB corresponding to mean DGD of 20 ps, 30 ps, 40 ps and 50 ps respectively at $h=0.75$. Further increase of DGD leads to a BER floor above 10^{-9} which can not be lowered by further increasing the signal power.

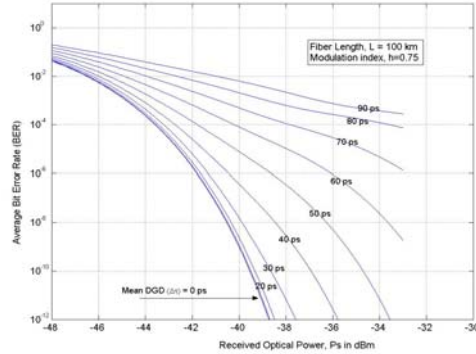


Fig. 1: Plots of average BER versus received power, Ps for heterodyne CPFSK receiver impaired by PMD at modulation index 0.75

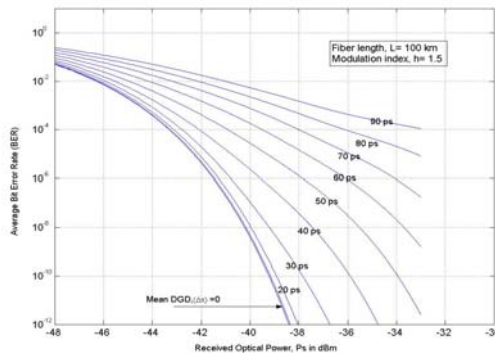


Fig.2: Plots of average BER versus received power,Ps for heterodyne CPFSK impaired by PMD at modulation index 1.50

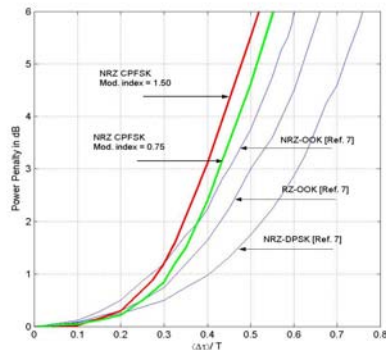


Fig.3: PMD-induced power penalty as a function of DGD/bit duration ($\langle \Delta \tau \rangle / T$) for NRZ- and RZ-OOK, NRZ-DPSK and heterodyne NRZ-CPFSK system.

It is noticed that BER floors occur at about 10^{-7} , 10^{-5} and 2×10^{-4} corresponding to mean DGD of 70

ps, 80 ps and 90 ps respectively at 10 Gb/s and modulation index of 0.75. The amount of penalty is approximately 5.50 dB when mean DGD is 50 ps at modulation index 1.50

The plots of penalty due to other modulation schemes as reported in [7] are also shown in Fig. 3 for comparison. It is further noticed that penalty suffered by heterodyne detection CPFSK with modulation index 0.75 is higher than that of NRZ-OOK system. But at higher modulation index, penalty suffered by heterodyne CPFSK is also higher than that of OOK and DPSK system.

V. CONCLUSION

An analytical approach is presented to evaluate the impact of polarization mode dispersion on the BER performance of heterodyne CPFSK system. The results show that the penalty due to PMD suffered by the heterodyne CPFSK system is significant at higher modulation index and higher values of the differential group delay between the two eigenmodes

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