# EFFECT OF POLARIZATION MODE DISPERSION ON DIRECT DETECTION CPFSK OPTICAL TRANSMISSION SYSTEM IN A SINGLE MODE FIBER

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**ABSTRACT**

An analytical approach is developed to evaluate the bit error rate (BER) performance degradation of an optical direct detection CPFSK system caused by signal phase distortion due to polarization mode dispersion (PMD) in a single mode fiber. The average BER performance results are evaluated at a bit rate of 10 Gb/s considering Maxwellian distribution for the differential group delay (DGD). The results show that the performance of optical direct detection system suffers power penalty of 0.15 dB, 1.0 dB, 3.40 dB and 6.40 dB corresponding to mean DGD of 20 ps, 40 ps, 60 ps and 80 ps respectively at a BER of $10^{-9}$ operating at 10 Gb/s and modulation index of 0.5. Furthermore, at increased values of the mean DGD there occur BER floors above $10^{-9}$, which cannot be lowered by further increasing the signal power. It is noticed that BER floors occur at about $10^{-4}$ and $10^{-8}$ for modulation index of 0.50 and 1.0 respectively to a corresponding mean DGD of 90 ps. The effect of PMD is found to be more detrimental at higher modulation index.

## 1. INTRODUCTION

Optical continuous-phase FSK (CPFSK) is an attractive modulation format as it allows generation of compact spectra that allows for receiver envelop detection by properly selecting the modulation index. However, with the advent of long distance high bit rate optical systems, PMD is one of the serious degradation factors limiting bit rate and the transmission distance, becomes especially important as the bit rate increases higher than 10 Gb/s. The PMD induces the differential group delay (DGD) between two principal polarization axes of the transmission media and distorts the signal waveform that results bit pattern corruption and higher bandwidth requirement, and eventually contributes to BER deterioration, performance variation or system fading even at moderate bit rate [1].

The effect of PMD has been the focus of research in fiber communications for more than a decade and the developed ideas are briefly described in a series of publication [1]-[8]. The effect for PMD induced degradation for several modulation formats in high-speed long haul optical transmission system is given for both compensated and uncompensated system [3]. An analytical analysis of PMD impairment of optical multilevel DPSK system in terms of outage probability as function of mean DGD and its mitigation at 40 Gb/s has been presented [4]. The simulation results on the effects of PMD on an amplified IM-DD system is reported in Ref. [5] as a function of the DGD. The effect of PMD on the bit error rate performance of heterodyne FSK system is also reported [7]. Recently, the performance of optical DPSK system with direct detection receiver is reported in presence of PMD [8].

In this paper, we provide an analytical approach to evaluate the BER performance limitations of an optical direct detection CPFSK system impaired by PMD. Here, we take the whole statistical picture into account by investigating the corresponding power penalty in terms of BER degradation, which is a very relevant measure of system performance. This method is based on the linear approximation of the output phase of a linearly filtered angle-modulated signal such as the CPFSK signal. The probability density function (pdf) of the random phase fluctuation due to PMD and group velocity dispersion (GVD) at the output of the receiver is determined analytically. Based on the pdf of the random phase fluctuation, a conditional BER expression is derived. The temporal behaviors of the fiber PMD are of statistical in nature due to randomness of the birefringence variations along the fiber structure. Therefore to assess accurately the optical transmission impairment due to PMD, we determine the average BER and power penalty for different mean values of DGD, considering Maxwellian probability density function for the DGD, at the receiver output in the presence of receiver noise. The bit error rate performance and power penalty suffered by the system due to PMD at a BER of $10^{-9}$ are evaluated at a bit rate of 10 Gb/s.

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## II. SYSTEM MODEL

The block diagram of an optical CPFSK transmission system with direct detection model is shown in fig.1. Fig.2 shows the low pass equivalent CPFSK system model considering PMD and chromatic dispersion. The signal components in the two output PSPs propagate independently through the entire system from the modulator to the balanced receiver. In the transmitter, non-return to zero (NRZ) data at 10 Gb/s is used to directly modulate a laser to generate the CPFSK signal that is transmitted through a single mode fiber. At the receiving end, MZI based direct detection receiver detects the received optical signal. The fields corresponding to the two PSPs are separately detected and the resulting photocurrents are summed. We decompose the entire system into a pair of subsystems, each corresponding to one PSP. We sum the outputs of the two subsystems to obtain sampled decision variables $i_m(t) = i_{m1}(t) + i_{m2}(t)$. The output photocurrent is filtered by a low pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero a binary '1' is detected, otherwise a binary '0' is detected.

## III. THEORETICAL ANALYSIS

The complex electric field at the output of the CPFSK transmitter and input to the fiber is represented as:

$$E_1(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_1 . e_1] \qquad (1)$$

$$E_2(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_2 . e_2] \qquad (2)$$

Where $f_c$ is the carrier frequency, $P_T$ is the transmitted optical power and the CPFSK modulating phase $\phi_s(t)$ is given by

$$\phi_s(t) = 2\pi\,\Delta f \int_{-\infty}^{t} \sum_{k=-\infty}^{\infty} a_k\, p(t - kT)\,dt + \phi_n(t) \qquad (3)$$

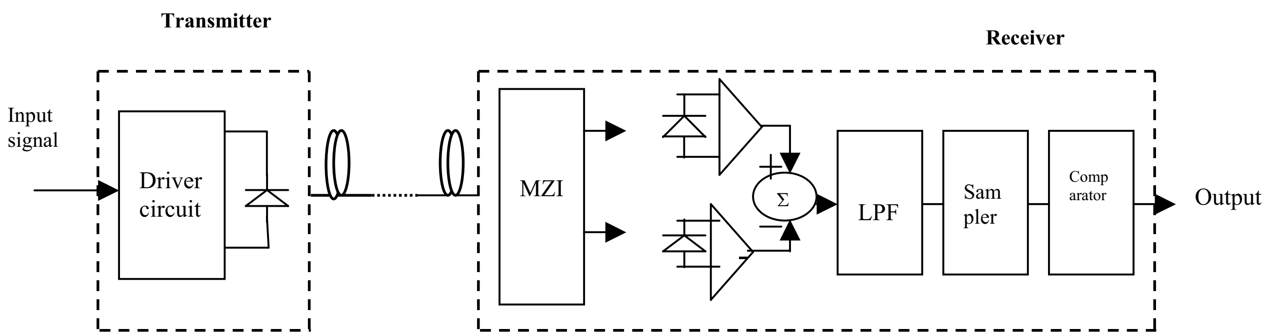


Fig.1: Block diagram of an optical CPFSK transmission system with MZI based direct detection receiver

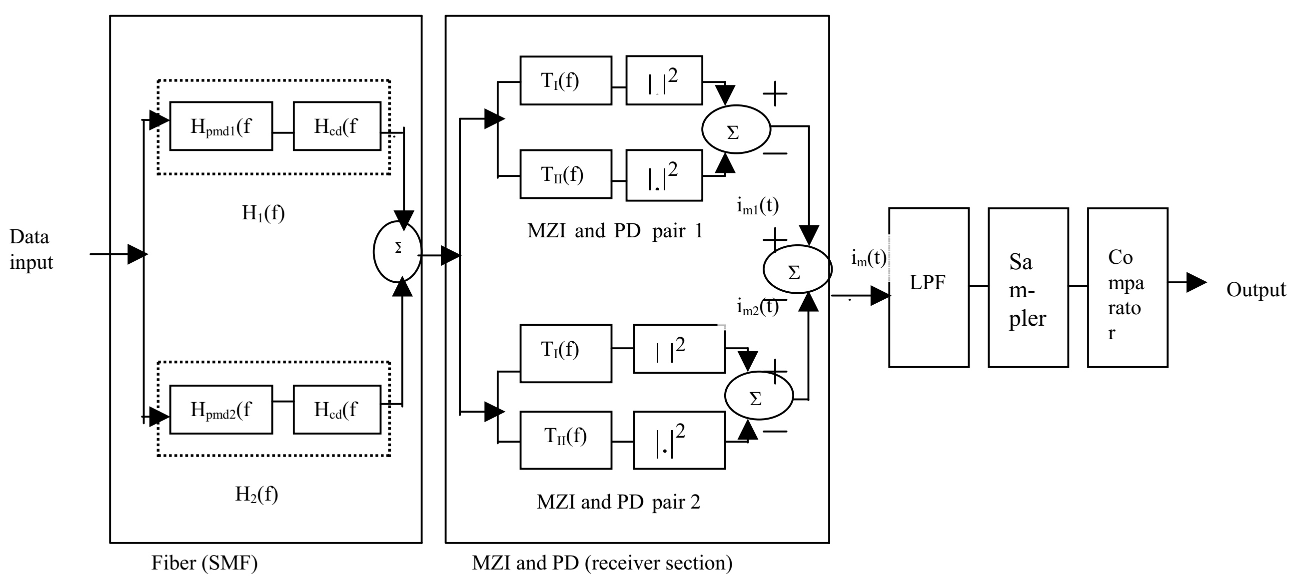


Fig.2: Low pass equivalent direct detection CPFSK system model considering PMD and chromatic dispersion

Where $a_k$ is the random NRZ bit, $\Delta f$ is the peak frequency deviation, $p(t)$ is the elementary pulse shape and $\phi_n(t)$ is the phase noise of the transmitting laser. Here $c_1, c_2$ represent unit vectors and $e_1, e_2$ represent the two principal states of polarization respectively. The electric field at the output of the fiber is then given by:

$$E_{01}(t) = \sqrt{2P_T}\ \exp[j\{2\pi f_c t + \phi_s(t)\}][c_1 . e_1] \otimes h_1(t) \quad (4)$$

$$E_{02}(t) = \sqrt{2P_T}\ \exp[j\{2\pi f_c t + \phi_s(t)\}][c_2 . e_2] \otimes h_2(t) \quad (5)$$

Where $\otimes$ denotes convolution, $h_1(t)$ and $h_2(t)$ are the inverse Fourier transform of the low pass equivalent transfer function of a non-dispersion shifted lossless fiber $H_1(f)$ and $H_2(f)$ respectively, which include the effect of polarization mode dispersion (PMD) and group velocity dispersion (GVD) and given by [14],

$$H_1(f) = \sqrt{\alpha}\ \exp[j2\pi f(-\frac{\Delta\tau}{2}) - j\gamma(\pi fT)^2] \quad (6)$$

$$H_2(f) = \sqrt{1-\alpha}\ \exp[j2\pi f(\frac{\Delta\tau}{2}) - j\gamma(\pi fT)^2] \quad (7)$$

Where $\Delta\tau$ represents the differential group delay (DGD) between the two principal states of polarization (PSP), $\alpha$ is the PMD power-splitting ration and $\gamma$ is the chromatic dispersion index. Here we assume that there is negligible polarization-dependent loss

Assuming linear phase approximation the output electric fields for the two polarization states can be written as

$$E_{01}(t) = \sqrt{2P_S}\ \exp[j\{2\pi f_c t + \phi_{s1}(t) + \theta_{n1}(t)\}][c_1 . e_1] \quad (8)$$

$$E_{02}(t) = \sqrt{2P_S}\ \exp[j\{2\pi f_c t + \phi_{s2}(t) + \theta_{n2}(t)\}][c_2 . e_2] \quad (9)$$

Where,

$$\phi_{s1}(t) = 2\pi\Delta f \int_{-\infty}^{t} \sum_k a_k \ \mathrm{Re}[p(t-kT) \otimes h_1(t)]dt \quad (10)$$

$$\phi_{s2}(t) = 2\pi\Delta f \int_{-\infty}^{t} \sum_k a_k \ \mathrm{Re}[p(t-kT) \otimes h_2(t)]dt \quad (11)$$

In the FSK direct detection receiver with Mach-Zehnder interferometer (MZI), the MZI act as an optical discriminator and differentially detect the 'mark' and 'space' of the received FSK signal, which are then directly fed to a pair of photo-detectors. The transmittances of the two branches of the MZI are $T_I(f)$ and $T_{II}(f)$, and τ is the time delay between the two branches.

For a 'mark' transmission, the current at the output of the photo-detectors can be expressed as:

$$i_{m1}(t) = R_d P_s \cos[2\pi f_c \tau + \Delta\phi_{s1}(t,\tau) + \Delta\theta_{n1}(t,\tau)][c_1 . e_1] \quad (12)$$

$$i_{m2}(t) = R_d P_s \cos[2\pi f_c \tau + \Delta\phi_{s2}(t,\tau) + \Delta\theta_{n2}(t,\tau)][c_2 . e_2] \quad (13)$$

Where

$$\Delta\phi_{s1}(t,\tau) = 2\pi\ \Delta f \int_{t-\tau}^{t} \sum_k a_k g_1(t-kT)\,dt \quad (14)$$

$$\Delta\phi_{s2}(t,\tau) = 2\pi\ \Delta f \int_{t-\tau}^{t} \sum_k a_k g_2(t-kT)\,dt \quad (15)$$

For a 'mark' transmission and ideal CPFSK demodulation condition (assuming NRZ data), we have $2\pi f_c\ \tau = (2n+1)\frac{\pi}{2}$, where n is an integer; and $2\pi\Delta f\ \tau = \frac{\pi}{2}$. Thus, the currents at the output of photodetector can be expressed as,

$$i_{m1}(t) = R_d P_s \cos[2\pi f_c \tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_1(t-kT) + \Delta\theta_{n1}(t,\tau)][c_1 . e_1] \quad (16)$$

$$i_{m2}(t) = R_d P_s \cos[2\pi f_c \tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_2(t-kT) + \Delta\theta_{n2}(t,\tau)][c_2 . e_2] \quad (17)$$

Where,

$$q_1(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_1(t)\,dt \ ; \quad g_1(t) = \mathrm{Re}[p(t) \otimes h_1(t) \otimes h_{LP}(t)]$$

$$q_2(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_2(t)\,dt \ ; \quad g_2(t) = \mathrm{Re}[p(t) \otimes h_2(t) \otimes h_{LP}(t)]$$

Where $h_{LP}(t)$ represents the impulse response of the electrical low pass filter, which is assumed to be fifth-order Bessel function type. The transfer function of the low pass filter is given by,

$$H_{LP}(f) = \frac{945}{jF^3 + 15F^4 - 105jF^3 - 420F^2 + 945jF + 945} \quad (18)$$

Where $F = 2.43 f / B_e$ and $B_e$ is the 3-db cutoff frequency.

The output phase distortion due to PMD and GVD is given by,

$$\Delta\phi_1(t,\tau,\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_1(t-kT) = \overline{\Delta\varphi}_{s1}(t,\tau) + \xi_1(t,\tau) \quad (19)$$

$$\Delta\phi_2(t,\tau.\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_2(t-kT) = \overline{\Delta\varphi}_{s2}(t,\tau) + \xi_2(t,\tau) \quad (20)$$

Where $\overline{\Delta\varphi}_{s1}(t,\tau)$ and $\overline{\Delta\varphi}_{s2}(t,\tau)$ are mean output phase error; $\xi_1(t,\tau)$ and $\xi_2(t,\tau)$ are the random output phase fluctuation due to ISI bit pattern caused by PMD and GVD.

The total signal current at the output of LPF corresponding to a 'mark' is given by,

$$i_m(t) = R_d P_s x_1(t) + R_d P_s x_2(t) \qquad (21)$$

Where, $x_1(t)$ and $x_1(t)$ describe the phase noise induced by the interferometer intensity due to random phase fluctuation and can be expressed as,

$$x_1(t) = \cos[\Delta\varphi_1(t)] \qquad (22)$$

$$x_2(t) = \cos[\Delta\varphi_2(t)] \qquad (23)$$

Where

$$\Delta\varphi_1(t) = \Delta\phi_1(t,\tau,\Delta\tau) + \Delta\varphi_n(t) \qquad (24)$$

$$\Delta\varphi_2(t) = \Delta\phi_2(t,\tau,\Delta\tau) + \Delta\varphi_n(t) \qquad (25)$$

The low-pass filter output for 'mark' transmission, at a sampling time *t* is,

$$i_m(t) = 2R_d P_s\, [\cos(\delta\varphi)\cos(\Delta\varphi)] + n(t) \qquad (26)$$

where $$\delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) - \Delta\phi_2(t,\tau,\Delta\tau)}{2} \qquad (27)$$

and $$\Delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) + \Delta\phi_2(t,\tau,\Delta\tau)}{2} \qquad (28)$$

If we consider the mean phase distortion only then the conditional bit error rate can be expressed as,

$$P(e \mid \overline{\delta\varphi},\ \overline{\Delta\varphi}) = \frac{1}{2}\, erfc\left( Q \big/ \sqrt{2} \right) \qquad (29)$$

$$\text{Where, } Q = \frac{2R_d P_s\, [\cos(\overline{\delta\varphi})\cos(\overline{\Delta\varphi})]}{\sigma_m} \qquad (30)$$

The contribution of the term (random phase fluctuation) $\xi_1(t,\tau)$ of (19) and $\xi_2(t,\tau)$ of (20) can be evaluated by finding the probability density function (pdf). Assuming $\{a_k\}$ are independent and identically distributed (IID) random variables, the characteristic function of $\xi_1$ and $\xi_2$ is given by,

$$\Phi_{\Delta\phi_1}(j\xi_1) = \prod_{i=1}^{\infty} \cos[\xi_1\, q_{1i}(t)] = \sum_{i=1}^{\infty} \frac{(j\xi_1)^{2i}}{(2i)!} M'_{2i} \qquad (31)$$

$$\Phi_{\Delta\phi_2}(j\xi_2) = \prod_{i=1}^{\infty} \cos[\xi_2\, q_{2i}(t)] = \sum_{i=1}^{\infty} \frac{(j\xi_2)^{2i}}{(2i)!} M''_{2i} \qquad (32)$$

Where, $q_{1i}(t) = \left| q_1(t - iT) \right|$, $q_{2i}(t) = \left| q_2(t - iT) \right|$, $M'_{2i}$ and $M''_{2i}$ are the even order moments of the characteristic function of random variable $\xi_1$ and $\xi_2$ respectively. By using the above two equations the pdf $P_{\delta\varphi}(\delta\varphi)$ of $\delta\varphi$ and the pdf $P_{\Delta\varphi}(\Delta\varphi)$ of $\Delta\varphi$ can be evaluated following Ref. [7].

For a given value of differential group delay (DGD) $\varDelta\tau$, the conditional BER can be expressed as:

$$BER(\varDelta\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(e \mid \overline{\delta\varphi}, \overline{\Delta\varphi})\, P_{\delta\varphi}(\delta\varphi)\, P_{\Delta\varphi}(\Delta\varphi)\, d(\delta\varphi)\, d(\Delta\varphi) \qquad (33)$$

The statistical properties of PMD have been experimentally and theoretically studied and found that the evolution of DGD at particular frequency over time yields a Maxwellian probability density function given by,

$$P_{\varDelta\tau}(\varDelta\tau) = \frac{32\,\varDelta\tau^2}{\pi^2\,\langle\varDelta\tau\rangle^3}\, \exp\left( -\frac{4\,\varDelta\tau^2}{\pi\,\langle\varDelta\tau\rangle^2} \right), \qquad \varDelta\tau > 0 \qquad (34)$$

Where $\langle\varDelta\tau\rangle$ is the quadratic mean of instantaneous DGD, $\varDelta\tau$. The average bit error probability can now be evaluated as,

$$BER = \int_{-\infty}^{\infty} BER(\varDelta\tau)\, P_{\varDelta\tau}(\varDelta\tau)\, d(\varDelta\tau) \qquad (35)$$

## IV. RESULTS AND DISCUSSION

Following the analytical approach outlined above, we evaluated the bit error rate performance result of an optical heterodyne CPFSK transmission system with delay demodulation for several values mean DGD at a bit rate of 10 Gb/s. The plots of average BER versus received power, Ps are shown in Fig. 3 and Fig.4 at modulation index of 0.50 and 1.0 respectively for different values of mean DGD at a bit rate of 10 Gb/s. The results show that the BER performance is more affected at higher values of mean DGD and at higher modulation index.

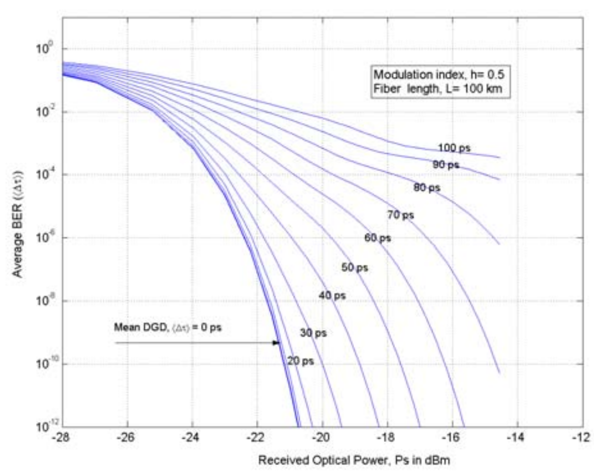


Fig. 3: Plots of average BER versus received power, Ps for direct detection CPFSK receiver impaired by PMD (modulation index h = 0.50)

CPFSK system at BER $10^{-9}$ is plotted in Fig.5 as a function of DGD normalized by bit duration. It is observed from Fig.3 that the penalty at BER = $10^{-9}$ is increasing with increasing values of the mean DGD. The amount of penalty is found to be approximately 0.15 db, 1.0 db, 3.40 db and 6.40 db corresponding to mean DGD of 20 ps, 40 ps, 60 ps and 80 ps respectively. Further increase of DGD leads to a BER floor above $10^{-9}$ which can not be lowered by further increasing the signal power. It is observed from Fig. 3 and Fig.4 that BER floors occur at about $10^{-4}$ and $10^{-8}$ for modulation index of 0.50 and 1.0 respectively to a corresponding mean DGD of 90 ps.

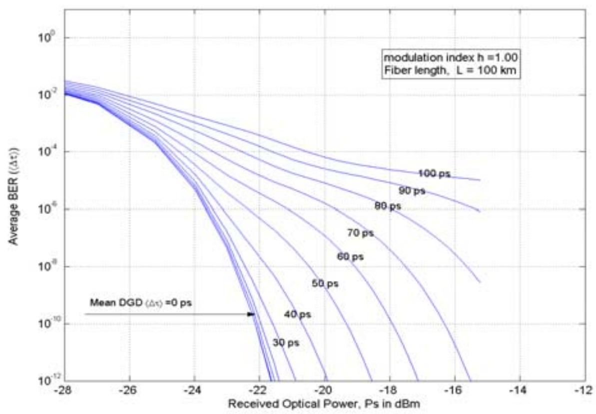


Fig. 4: Plots of average BER versus received power, Ps for direct detection CPFSK receiver impaired by PMD (modulation index, h = 1.0)

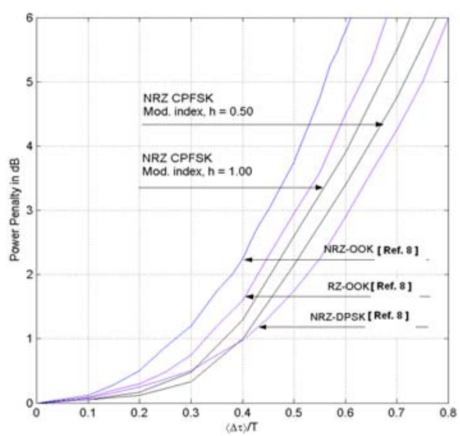


Fig.5: PMD-induced power penalty as a function of DGD/bit duration ( $< \Delta\tau > / T$ ) for NRZ- and RZ-OOK, NRZ-DPSK and direct detection NRZ-CPFSK system.

The plots of penalty due to other modulation schemes as reported in [8] are also shown in Fig. 5 for comparison. It is further noticed that penalty suffered by direct detection CPFSK with modulation index 0.50 and 1.0 is higher than that of NRZ-OOK system, but lower than RZ-OOK and NRZ-OOK system. For example, the penalty is 1.30 db and 2.70 db corresponding to DGD/bit duration ratio ( $\langle \Delta\tau \rangle / T$ ) are 0.40 and 0.50 respectively for direct detection CPFSK system at modulation index 1.0. The corresponding penalty values are 2.30 db, 1.70 db and 2.90 db, 3.80 db for NRZ-OOK and RZ-OOK respectively.

## V. CONCLUSION

An analytical approach is presented to evaluate the impact of polarization mode dispersion on the BER performance of a direct detection CPFSK system. The results show that the penalty due to PMD suffered by the direct detection CPFSK system is 4.0 db corresponding to a mean DGD of 60 ps at modulation index of 1.0. Thus the amount of power penalty is higher at higher values of modulation index and higher values of the differential group delay between the two polarization axes. Also a direct-detection CPFSK system with modulation index of 0.50 suffers slightly higher amount of power penalty than a direct detection NRZ-DPSK system.

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