

Performance Limitations of an Optical IM-DD Transmission System Due to Polarization Mode Dispersion

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Abstract

An analytical approach is presented to evaluate the impact of polarization mode dispersion on the average bit error rate (BER) performance of a intensity modulated direct detection (IM-DD) optical transmission system considering Maxwellian distribution for the differential group delay (DGD). The results show that the performance of an IM-DD system suffers power penalty of 0.27 dB, 0.45 dB and 2.4 dB corresponding to mean DGD of 20 ps, 35 ps and 40 ps respectively at a BER of 10^{-9} operating at a bit rate of 10 Gb/s. Furthermore, at increased values of the mean DGD there occur BER floors above 10^{-9} which can not be lowered by further increasing the signal power. It is noticed that BER floors occur at about 2×10^{-9} , 10^{-7} and 10^{-5} corresponding to mean DGD of 45 ps, 50 ps and 60 ps respectively at 10Gb/s. The effect of PMD is found to be more detrimental at higher bit rates.

1. Introduction

In a real single mode fiber (SMF) the propagating light is split into two local polarization modes that travel with different velocities. This effect is known as polarization mode dispersion (PMD) and it causes pulse spreading in digital system and distortion in analog systems. Thus PMD in SM fibers imposes a potential limit to high capacity long distance optical communication systems, particularly for systems operating at zero chromatic dispersion wavelengths. PMD contributes to BER deterioration, performance variation or system fading even at moderate bit rate [1].

The effect of PMD has been the subject of considerable research interest during the past few

years. Some studies treated PMD as deterministic term in numerical simulation or are based on simple analytical approach of determining the conditional bit error probability [2]. The effect of PMD on the BER performance is theoretically assessed at a bit rate of 2.5 Gb/s for an IM-DD system [3]. The results are reported in terms of receiver sensitivity penalty at a given BER conditioned for a given value of the DGD. The simulation results on the effect of PMD on an amplified IM-DD system are also reported conditioned on a given value of DGD in [4].

In this paper, we provide an analytical approach to evaluate unconditioned BER performance limitations of an optical IM-DD transmission system impaired by PMD. Since temporal behaviors of fiber PMD are of statistical nature as a result of the randomness of the birefringence variations along the fiber structure, therefore to determine more accurately the optical transmission impairment due to PMD we incorporate the PMD statistics. In this paper, we determine the unconditional BER and power penalty analytically considering the DGD to have a Maxwellian probability density function [7].

2. System Model

The schematic configuration of an optical transmission system model considered in the analysis is shown in Fig.1. A DFB or DBR laser may be used at the transmitter. The pseudorandom data pattern at a rate of 10 Gb/s is used to intensity modulate the laser source and output fed to the SM fiber. The optical signal is received by a direct detection receiver with a PIN photodiode. The output of the pre-amplifier is then filtered by a low-pass filter and fed to a sampler followed by a comparator.

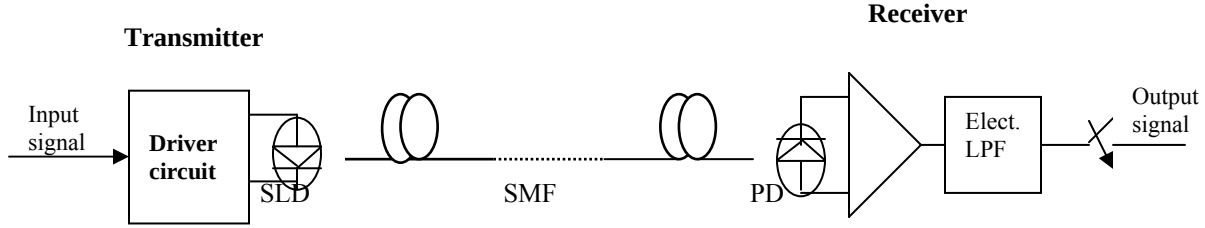


Fig.1: Schematic configuration of an optical transmission system model

3. Theoretical Analysis

A very elegant way to study PMD is based on the model of the principal states of polarization (PSP). For a given fiber, at a fixed time and optical frequency, there always exist two polarization states, called PSP. When operating in a quasi-monochromatic regime, the output PSPs are the two orthogonal output states of polarization for which the output polarization does not depend on the optical frequency at first order. The corresponding orthogonal input polarization states are called the input PSPs.

The difference in arrival time $\Delta\tau$ between the PSPs is called differential group delay (DGD) and it is the cause of pulse broadening at the output of a fiber when energy is split between the two PSPs at the input. If $\Delta\tau$ is the DGD between the PSP's, T is the bit interval, T_0 is the FWHM of the input pulse and 2θ is the angle between the Stokes vector representing the state of polarization of the input pulse and of the input PSP's, the output pulse width can be given by [3]

$$\sqrt{T^2 - T_0^2} = \frac{1}{2} \Delta\tau \sin(2\theta) \quad (1)$$

Generally for commercial SM fibers, PMD becomes the only serious consideration when an optical transmission system with a high bit rate distance product is operating in the wavelength where the fiber chromatic dispersion is negligible. Due to the more mature EDFA technology at the 1.55 μm wavelength window, PMD is relevant for system using dispersion-shifted fibers. Therefore, we assume dispersion-shifted fiber as a transmission medium. Fiber nonlinearities and optical amplifier related polarization sensitivities are also ignored in this paper.

The optical field $E_t(t)$ coupled into the fiber can be modeled as

$$E_t(t) = \sqrt{P_t} \sum_{k=-\infty}^{\infty} b_k g(t - kT) (c_1 e_1 + c_2 e_2) \quad (2)$$

where P_t is the transmitted peak power, $b_k = 0, 1$ represents the transmitted bit, and $g(t)$ the transmitted

pulse shape. The vectors represent the input PSP's are indicated with

$$e_i = (e_{ix} x + e_{iy} y) \quad (i = 1, 2) \quad (3)$$

and the complex coefficients c_1 and c_2 determine the SOP of the transmitted field ($|c_1|^2 + |c_2|^2 = 1$).

In deriving the output electric field we assume that the signal bandwidth is much smaller than the PSP's bandwidth. This constraint is fulfilled for all the practical values of the bit rate if a standard DFB or DBR laser is used at the transmitter. Neglecting chromatic dispersion, in a reference frame solid to the output PSP's, the received field $E(t)$ is given by [5]

$$E(t) = \sqrt{P} \sum_{k=-\infty}^{\infty} b_k [\cos \theta g(t - kT) x + \sin \theta g(t - kT - \Delta\tau) e^{-i\varphi} y] \quad (4)$$

where P is the received peak power, θ is the angle between the electric field representing the state of polarization (SOP) of the transmitted pulse and the direction of the input PSPs and φ is the fiber induced phase difference between the two waves.

At the receiver, after detection by means of a photodiode and filtering by a baseband filter of pulse response $h(t)$, the photocurrent $j(t)$ taking into account the orthogonal property of the PSP's, the k th decision variable is given by [3]

$$j_k = R_p PM \{ [\cos^2 \theta \gamma(\Delta\tau \sin^2 \theta) + \sin^2 \theta \gamma(\Delta\tau \sin^2 \theta - \Delta\tau)] + R_p PM \sum_{h \neq k} h_k [\cos^2 \theta \gamma(t_k - hT) + \sin^2 \theta \gamma(t_k - hT - \Delta\tau)] + n(t_k) \} \quad (5)$$

Where R_p represents the photodiode responsivity, M the photodiode average gain if an APD is used (if a PIN is used $M=1$), $t_k = (kT + \Delta\tau \sin^2 \theta)$, sampling instant $n(t_k)$ the noise sample and

$$\gamma(t) = \int_{-\infty}^{\infty} h(t - \xi) |g(\xi)|^2 d\xi \quad (6)$$

The noise power σ_k^2 , which depends on the received signal can be easily evaluated assuming the dark current noise is negligible and according to [6] it is expressed as

$$\sigma_k^2 = \sigma_T^2 + 2e < j_k > BM^{2+\xi} \quad (7)$$

where σ_T^2 is the receiver thermal noise power in the signal bandwidth, ξ is a parameter typically of the adopted photodiode and $\langle * \rangle$ indicates the ensemble average for a fixed transmitted message, B is the bit rate.

The probability density function of the thermal and shot noise at the receiver output are well described by Gaussian statistics with zero mean and variance equal to the respective noise power at the output. Bit error occurs when a '1' is transmitted, but '0' is received and when a '0' is transmitted but '1' is received.

We can express the BER conditioned on a given value of e DGD, $\Delta\tau$ by

$$BER(\Delta\tau) = P(1)P(0/1) + P(0)P(1/0) \quad (8)$$

where $P(1)$ probability of transmission of '1', $P(0/1)$ probability of falsely identifying a binary '1' as '0', $P(0)$ probability of transmission of '0' and $P(1/0)$ probability of falsely identifying a binary '0' as '1'. From (5) it is evident that the main effect of polarization mode dispersion is to generate intersymbol interference (ISI). We represent this term by

$$i_{isi} = R_p PM \sum_{h \neq k} b_k [\cos^2 \theta \gamma(t_k - hT) + \sin^2 \theta \gamma(t_k - hT - \Delta\tau)] + n(t_k) \quad (9)$$

The effect of ISI can be evaluated by approximating the (9) with terms $h = k-1, k-2, k-3, k-4$ and $h = k+1, k+2, k+3, k+4$, that is considering in a fourth approximation only the interference among the adjacent bits.

The probability that a '0' is received when a '1' is transmitted and the probability that '1' is transmitted when a '0' is transmitted can be given by

$$p(0/1) = 0.5 \operatorname{erfc} \left(\frac{(\langle j_k \rangle - \langle i_{isi} \rangle) - j_{th}}{\sqrt{2} \sigma_i^2} \right) \quad (10)$$

$$p(1/0) = 0.5 \operatorname{erfc} \left(\frac{\langle i_{isi} \rangle - j_{th}}{\sqrt{2} \sigma_i^2} \right) \quad (11)$$

where j_{th} is the optimum threshold value and σ_i the noise sample standard deviation corresponding to the considered pattern. Of course the value of j_{th} depends on different conditions as pulse shape, noise, filter shape and so on.

Now considering the SM fiber as a binary symmetric channel that indicates equal probability of occurrence for a '0' and a '1' bit (i.e., $P(0)=P(1)=0.50$). Using the (10) and (11) in (8) we can express the BER conditioned on a given value of $\Delta\tau$ as,

$$BER(\Delta\tau) = 0.25 \left(\operatorname{erfc} \left(\frac{(\langle j_k \rangle - \langle i_{isi} \rangle) - j_{th}}{\sqrt{2} \sigma_i^2} \right) + \operatorname{erfc} \left(\frac{\langle i_{isi} \rangle - j_{th}}{\sqrt{2} \sigma_i^2} \right) \right) \quad (12)$$

Equation (8)- (12) allow the system conditional BER to be evaluated if the system parameters, as the transmitted pulse shape, the transmitted SOP are known.

Even if the fiber is perfectly circular, external mechanical or thermal stresses cause small asymmetric to the fiber core. The temporal behavior of the PMD can be due to relatively fast changes in the environment such as ambient temperature and local vibration, or slow change such as aging. Thus DGD changes with time due to external stress. Beyond the birefringence correlation length, the polarization axes of the fiber are uncorrelated. It is shown that if the fiber length is much longer than the correlation length of the disturbances that cause the changes of symmetry in fiber geometry and stress, the DGD $\Delta\tau$ between the two PSP's follows a Maxwellian probability density function [7]. With a mean DGD of $\langle \Delta\tau \rangle$, Maxwellian distribution of the instantaneous DGD is given by,

$$P_{\Delta\tau}(\Delta\tau) = \frac{32 \Delta\tau^2}{\pi^2 \langle \Delta\tau \rangle^3} \exp \left(-\frac{4 \Delta\tau^2}{\pi \langle \Delta\tau \rangle^2} \right), \quad \Delta\tau > 0 \quad (13)$$

The average bit error probability can now be evaluated as,

$$BER = \int_{-\infty}^{\infty} BER(\Delta\tau) P_{\Delta\tau}(\Delta\tau) d(\Delta\tau) \quad (14)$$

4. Results and Discussion

Following the analytical approach, the bit error rate performance results for conditional case where the input power is equally divided among the PSP's according to (3), are evaluated at a bit rate of 10 Gb/s with several values of instantaneous DGD. The evaluation is performed assuming $\xi = 1$ and $M=1$ for the PIN photodiode and supposing that the electrical pulse has a Gaussian shape.

The plots for conditional BER versus received optical power P_s are shown in Fig. 2 for the DGD of 0, 40 ps, 60 ps, 80 ps, 100 ps, 120 ps, 140 ps and 160 ps respectively. The results show that the BER is highly degraded when the DGD is higher for a given fiber length and the system suffers a significant amount of power penalty due to the effect of PMD. It is observed that the penalty at given BER is increasing with increasing value of the DGD. Penalty is found to be approximately 0.30 dB, 0.55 dB, 1.0 dB, 2.05 dB, 3.90 dB and 7.75 dB for DGD of 40 ps, 80 ps, 100 ps, 120 ps, 140 ps and 160 ps respectively.

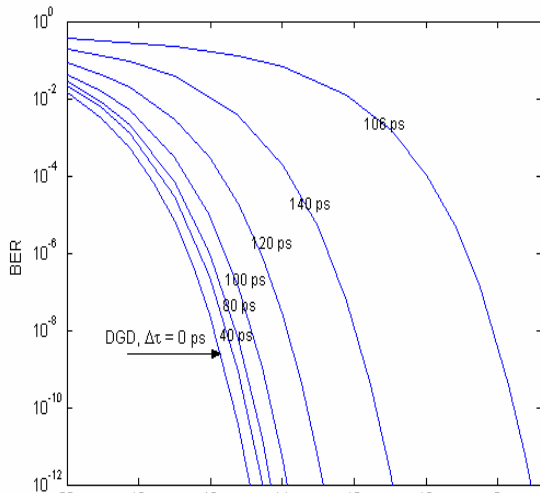


Fig. 2: Plots of conditional BER versus received optical power, P_s , for a 10 Gb/s system impaired by PMD

The plots for average BER versus received optical power P_s are shown in Fig. 3 for 10 Gb/s and different values of mean DGD. The results show that the BER is highly degraded when the mean DGD is higher for a given fiber length and the system suffers a significant amount of power penalty due to the effect of PMD. It is observed that the penalty at $\text{BER}=10^{-9}$ is increasing with increasing value of the mean DGD. Penalty is found to be approximately 0.27dB, 0.38 dB 0.45 dB and 2.4 dB for mean DGD 20 ps, 30ps, 35 ps and 40 ps respectively. Further increase of DGD leads to a BER floor above 10^{-9} .

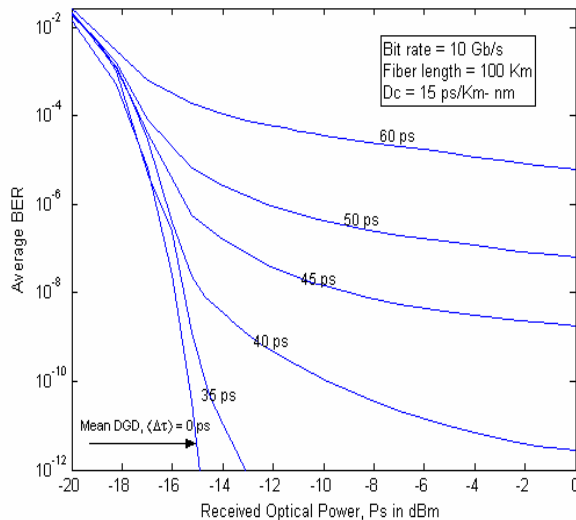


Fig.3: Plots of unconditional BER versus received optical power P_s , for a 10 Gb/s system impaired by PMD

5. Conclusion

In this paper an analytical method is presented to evaluate the impact of PMD on the BER performance of an IM-DD systems on the BER performance. BER performance results are evaluated for a range of values of DGD. The results show that the penalty due to PMD suffered by an IM-DD system is significant at higher DGD between the two polarization modes.

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