Performance Evaluation of an Optical Direct Detection CPFSK Transmission System Affected by Polarization Mode Dispersion

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***Abstract*- The effect of polarization mode dispersion (PMD) imposes serious restriction on the allowable distance and ultimate hurdle for the massive deployment of multi-gigabit transmission rate in an optical communication system. An analytical approach is presented to determine the impact of signal phase distortion due to PMD in a single mode fiber (SMF) on the bit error rate (BER) performance of an optical continuous phase FSK (CPFSK) transmission system with MZI based direct detection receiver. The probability density function (pdf) of the random phase fluctuation due to PMD and group velocity dispersion (GVD) at the output of the receiver is determined analytically. Based on the pdf of the random phase fluctuation the BER performance results are evaluated at a bit rate of 10 Gb/s fiber with dispersion co-efficient Dc = 15 ps/km-nm for different values of the mean differential group delay (DGD). The computed results show that the direct detection CPFSK system suffers a significant amount of power penalty due the effect of PMD at a bit rate of 10 Gb/s. At a modulation index of 0.5, the penalty at a BER of 10^{-9} is found to be 0.5 dB, 1.25 dB, 2.35 dB and 4.0 dB when the mean differential group delay (DGD) is 10 ps, 20 ps, 30 ps and 50 ps respectively for a fiber length of 100 km. At modulation index of 1.0, the penalty is approximately 5.25 dB for a mean DGD of 40 ps. The penalty is more significant at higher modulation index and bit rate.**

***Index Terms*: Polarization mode dispersion, Bit error rate, direct detection, Differential group delay, probability density function.**

A. Introduction

Optical continuous-phase FSK (CPFSK) is an attractive modulation format as it allows generation of compact spectra that allows for receiver envelop detection by properly selecting the modulation index [1-2]. However, at high bit rate operation of such systems in single-mode fiber, the major limitation is polarization mode dispersion. As the two polarization fields moves at different group velocities with a differential group delay (DGD), there is a fluctuation in signal phase between the two principal states of polarization which causes a random phase fluctuation of the signal itself resulting in spectral broadening of the transmitted signal and higher bandwidth requirement and contributes to BER deterioration, performance variation or system fading even at moderate bit rate [1-4].

The effect of PMD has been the subject of considerable research interests during the past few years. The effect for PMD on the error probability is theoretically assessed at a bit rate of 2.5 Gb/s for an IM-DD system [4]. Some studies treated PMD as deterministic term in numerical solution or are based on simple analytical approach of determining the conditional bit error probability [5]. The simulation results on the effects of PMD on an amplified IM-DD system is reported in Ref. [6] as a function of the DGD. The effect of PMD on the bit error rate performance of heterodyne FSK system is also reported [7]. Recently, the performance of optical DPSK system with direct detection receiver is reported in presence of PMD [8].

In this paper, we provide an analytical approach to evaluate the BER performance limitations of an optical direct detection CPFSK system impaired by PMD. The method is based on the linear approximation of the output phase of a linearly filtered angle-modulated signal such as the CPFSK signal. The expression for the overall degradation in signal phase is derived following an analytical approach by using an equivalent transfer function of a single fiber which includes the effects of PMD in presence of group velocity dispersion (GVD). The conditional BER is then determined using the pdf's of the random phase fluctuation due to PMD and GVD at the receiver output in the presence of receiver noise for different values of mean DGD. Power penalty suffered by the system due to PMD is evaluated at a bit rate of 10 Gb/s.

B. System Model

The block diagram of an optical CPFSK transmission system with direct detection model is shown in Fig.1. Fig.2 shows the low pass equivalent CPFSK system model considering PMD and chromatic dispersion. In the transmitter, non-return to zero (NRZ) data at 10 Gb/s is used to directly modulate a laser to generate the CPFSK signal that is transmitted through a single mode fiber. At the receiving end, MZI based direct detection receiver detects the received optical signal. The output from the two photodiodes are combined and filtered by a low pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero a binary '1' is detected, otherwise a binary '0' is detected.

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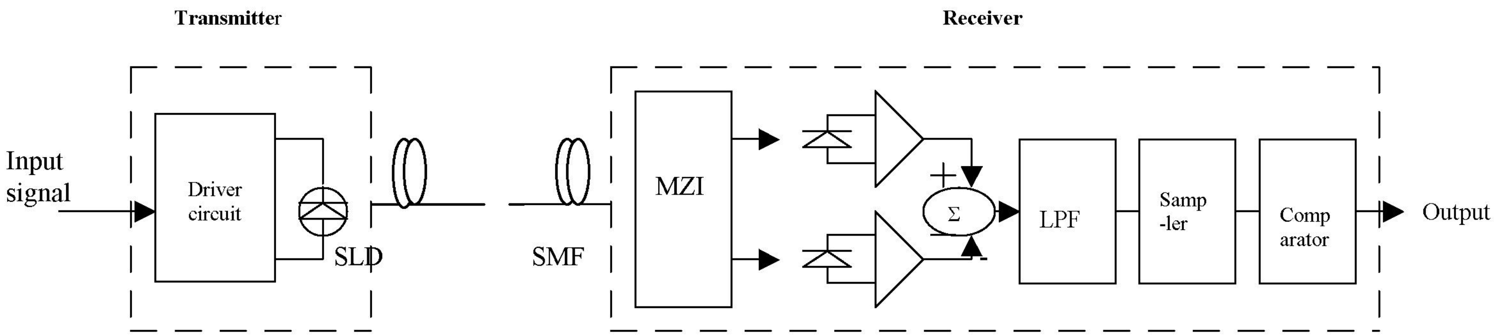

Fig.1: Block diagram of an optical CPFSK transmission system with MZI based direct detection receiver

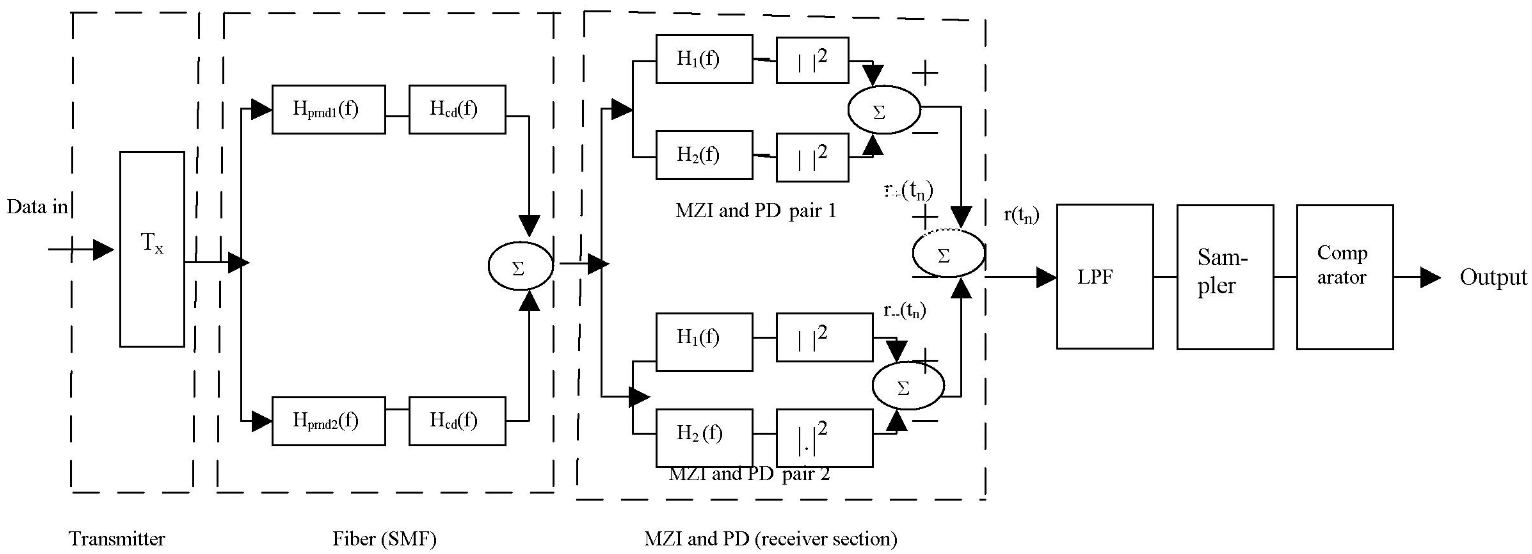

Fig.2: Low pass equivalent direct detection CPFSK system model considering PMD and chromatic dispersion

C. Theoretical Analysis

The complex electric field at the output of the CPFSK transmitter and input to the fiber is represented as:

$$E_1(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_1.e_1] \qquad (1)$$

$$E_2(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_2.e_2] \qquad (2)$$

where f_c is the carrier frequency, P_T is the transmitted optical power and the CPFSK modulating phase $\phi_s(t)$ is given by

$$\phi_s(t) = 2\pi\,\Delta f \int_{-\infty}^{t} \sum_{k=-\infty}^{\infty} a_k\, p(t-kT)dt + \phi_n(t) \qquad (3)$$

where a_k is the random NRZ bit, Δf is the peak frequency deviation, $p(t)$ is the elementary pulse shape and $\phi_n(t)$ is the phase noise of the transmitting laser. Here c_1, c_2 represent unit vectors and e_1, e_2 represent the two principal states of polarization respectively. The electric field at the output of the fiber is then given by:

$$E_{01}(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_1.e_1] \otimes h_1(t) \qquad (4)$$

$$E_{02}(t) = \sqrt{2P_T}\,\exp[j\{2\pi f_c t + \phi_s(t)\}][c_2.e_2] \otimes h_2(t) \qquad (5)$$

where $h_1(t)$ and $h_2(t)$ are the inverse Fourier transform of the fiber transfer function $H_1(f)$ and $H_2(f)$ respectively which include the effect of polarization mode dispersion and group velocity dispersion (GVD) and $\otimes$ denotes convolution. The low pass equivalent expression for $H_1(f)$ and $H_2(f)$ are given by [7]

$$H_1(f) = \sqrt{\alpha}\,\exp[j2\pi f(-\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \qquad (6)$$

$$H_2(f) = \sqrt{1-\alpha}\ \exp[\ j2\pi\ f(\frac{\Delta\tau}{2}) - j\gamma(\pi fT)^2\ ] \quad (7)$$

where $\Delta\tau$ represents the differential group delay (DGD) between the two principal states of polarization (PSP), α is the PMD power-splitting ration and γ is the chromatic dispersion index.

Assuming linear phase approximation the output electric fields for the two polarization states can be written as

$$E_{01}(t) = \sqrt{2P_s}\ \exp[j\{2\pi f_c t + \phi_{s1}(t) + \theta_{n1}(t)\}][c_1.e_1] \quad (8)$$

$$E_{02}(t) = \sqrt{2P_s}\ \exp[j\{2\pi f_c t + \phi_{s2}(t) + \theta_{n2}(t)\}][c_2.e_2] \quad (9)$$

where,

$$\phi_{s1}(t) = \mathrm{Re}[\phi_s(t) \otimes h_1(t)]$$
$$= 2\pi\ \Delta f \int_{-\infty}^{t} \sum_k a_k\ \mathrm{Re}[p(t-kT) \otimes h_1(t)]dt \quad (10)$$

$$\phi_{s2}(t) = \mathrm{Re}[\phi_s(t) \otimes h_2(t)]$$
$$= 2\pi\ \Delta f \int_{-\infty}^{t} \sum_k a_k\ \mathrm{Re}[p(t-kT) \otimes h_2(t)]dt \quad (11)$$

In the FSK direct detection receiver with Mach-Zehnder interferometer (MZI), the MZI act as an optical discriminator and differentially detect the 'mark' and 'space' of the received FSK signal, which are then directly fed to a pair of photo-detectors.

For a 'mark' transmission, the current at the output of the photo-detectors can be expressed as:

$$i_{m1}(t) = R_d P_s \cos[\ 2\pi f_c\tau \quad + \Delta\phi_{s1}(t,\tau) + \Delta\theta_{n1}(t,\tau)][c_1.e_1] \quad (12)$$

$$i_{m2}(t) = R_d P_s \cos[2\pi f_c\tau + \Delta\phi_{s2}(t,\tau) + \Delta\theta_{n2}(t,\tau)][c_2.e_2] \quad (13$$

where

$$\Delta\phi_{s1}(t,\tau) = \phi_{s1}(t) - \phi_{s1}(t-\tau)$$
$$= 2\pi\ \Delta f \int_{t-\tau}^{t} \sum_k a_k g_1(t-kT)dt \quad (14)$$

$$\Delta\phi_{s2}(t,\tau) = \phi_{s2}(t) - \phi_{s2}(t-\tau)$$
$$\bullet \quad = 2\pi\ \Delta f \int_{t-\tau}^{t} \sum_k a_k g_2(t-kT)dt \quad (15)$$

τ is the time delay between the two branches of the MZI. For a 'mark' transmission and ideal CPFSK demodulation condition (assuming NRZ data), we have $2\pi f_c\ \tau = (2n+1)\frac{\pi}{2}$, where n is an integer; and $2\pi\ \Delta f\ \tau = \frac{\pi}{2}$. Thus, the currents at the output of photodetector can be expressed as,

$$i_{m1}(t) = R_d P_s \cos[\ 2\pi f_c\tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_1(t) +$$
$$\frac{\pi}{2} \sum_{k\neq 0} a_k q_1(t-kT) + \Delta\theta_{n1}(t,\tau)][c_1.e_1] \quad (16)$$

$$i_{m2}(t) = R_d P_s \cos[\ 2\pi f_c\tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_2(t) +$$
$$\frac{\pi}{2} \sum_{k\neq 0} a_k q_2(t-kT) + \Delta\theta_{n2}(t,\tau)][c_2.e_2] \quad (17)$$

where, $q_1(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_1(t)dt$; $g_1(t) = \mathrm{Re}[p(t) \otimes h_1(t)]$

and $q_2(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_2(t)dt$; $g_2(t) = \mathrm{Re}[p(t) \otimes h_2(t)]$

The output phase distortion due to PMD and GVD is given by,

$$\Delta\phi_1(t,\tau,\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2} \sum_{k\neq 0} a_k q_1(t-kT)$$
$$= \overline{\Delta\varphi}_{s1}(t,\tau) + \xi_1(t,\tau) \quad (18)$$

$$\Delta\phi_2(t,\tau.\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2} \sum_{k\neq 0} a_k q_2(t-kT)$$
$$= \overline{\Delta\varphi}_{s2}(t,\tau) + \xi_2(t,\tau) \quad (19)$$

where $\overline{\Delta\varphi}_{s1}(t,\tau)$ and $\overline{\Delta\varphi}_{s2}(t,\tau)$ are mean output phase error; $\xi_1(t,\tau)$ and $\xi_2(t,\tau)$ are the random output phase fluctuation due to ISI bit pattern caused by PMD and GVD.

The total signal current at the output of LPF corresponding to a 'mark' is given by,

$$i_m(t) = R_d P_s x_1(t) + R_d P_s x_2(t) \quad (20)$$

where, $x_1(t)$ and $x_1(t)$ describe the phase noise induced by the interferometer intensity due to random phase fluctuation and can be expressed as,

$$x_1(t) = \cos[\Delta\varphi_1(t)] \quad (21)$$

$$x_2(t) = \cos[\Delta\varphi_2(t)] \quad (22)$$

where

$$\Delta\varphi_1(t) = \Delta\phi_1(t,\tau,\Delta\tau) + \Delta\varphi_n(t) \quad (23)$$

$$\Delta\varphi_2(t) = \Delta\phi_2(t,\tau,\Delta\tau) + \Delta\varphi_n(t) \quad (24)$$

Now ignoring the low pass filtering effect on the phase noise dependent signal term, for analytical simplicity, the low-pass filter output for 'mark' transmitted, at a sampling time *t* is,

$$i_m(t) \quad = 2R_d P_s\ [\cos(\delta\varphi)\cos(\Delta\varphi)] + n(t) \quad (25)$$

where $$\delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) - \Delta\phi_2(t,\tau,\Delta\tau)}{2} \quad (26)$$

and $$\Delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) + \Delta\phi_2(t,\tau,\Delta\tau)}{2} \quad (27)$$

If we consider the mean phase distortion only then the conditional bit error rate can be expressed as,

$$P(e \mid \overline{\delta\varphi},\ \overline{\Delta\varphi}) = \frac{1}{2}\, erfc\left(Q\big/\sqrt{2}\right) \qquad (28)$$

where, $$Q = \frac{2R_d P_s\ [\cos(\overline{\delta\varphi})\cos(\overline{\Delta\varphi})]}{\sigma_m} \qquad (29)$$

The contribution of the term (random phase fluctuation) $\xi_1(t,\tau)$ of (18) and $\xi_2(t,\tau)$ of (19) can be evaluated by finding the probability density function (pdf). Assuming $\{a_k\}$ are independent and identically distributed (IID) random variables, the characteristic function of ξ_1 and ξ_2 is given by,

$$\Phi_{\Delta\phi_1}(j\xi_1) = \prod_{i=1}^{\infty}\cos[\xi_1 q_{1i}(t)] = \sum_{i=1}^{\infty}\frac{(j\xi_1)^{2i}}{(2i)!}M'_{2i} \qquad (30)$$

$$\Phi_{\Delta\phi_2}(j\xi_2) = \prod_{i=1}^{\infty}\cos[\xi_2 q_{2i}(t)] = \sum_{i=1}^{\infty}\frac{(j\xi_2)^{2i}}{(2i)!}M''_{2i} \qquad (31)$$

where, $q_{1i}(t) = \left|q_1(t-iT)\right|$, $q_{2i}(t) = \left|q_2(t-iT)\right|$, M'_{2i} and M''_{2i} are the even order moments of the characteristic function of random variable ξ_1 and ξ_2 respectively. By using the above two equations the pdf $P_{\delta\varphi}(\delta\varphi)$ of $\delta\varphi$ and the pdf $P_{\Delta\varphi}(\Delta\varphi)$ of $\Delta\varphi$ can be evaluated following Ref. [7].

For a given value of mean differential group delay (DGD) $<\Delta\tau>$, the conditional BER can be expressed as:

$$BER(<\Delta\tau>) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(e \mid \overline{\delta\varphi}, \overline{\Delta\varphi})\, P_{\delta\varphi}(\delta\varphi)\, P_{\Delta\varphi}(\Delta\varphi)\, d(\delta\varphi)\, d(\Delta\varphi) \qquad (32)$$

The average error probability can be evaluated as

$$BER = \int_{-\infty}^{\infty} BER(<\Delta\tau>) P_{\Delta\tau}(\Delta\tau) d(\Delta\tau) \qquad (33)$$

where, $P_{\Delta\tau}(\Delta\tau)$ is the pdf of $\Delta\tau$ which has Maxwellian distribution with $\Delta\tau_m$ as quadratic mean of $\Delta\tau$ [6].

D. Results And Discussion

Following the analytical approach, the bit error rate performance results for direct detection CPFSK receiver without post detection low pass filter are evaluated at a bit rate of 10 Gb/s with several values of mean DGD and fiber lengths. The plots of conditional BER versus received power Ps are shown in Fig. 3 for mean DGD of 10 ps, 20 ps, 30 ps, 40 ps, 50ps and 60 ps with fiber length of 100 km and modulation index of 0.50 and 1.0.

The results show that the BER is highly degraded when the mean DGD is higher for a given fiber length and the system suffers a significant amount power penalty at BER = 10^{-9} which is increasing with increasing value of the mean DGD.

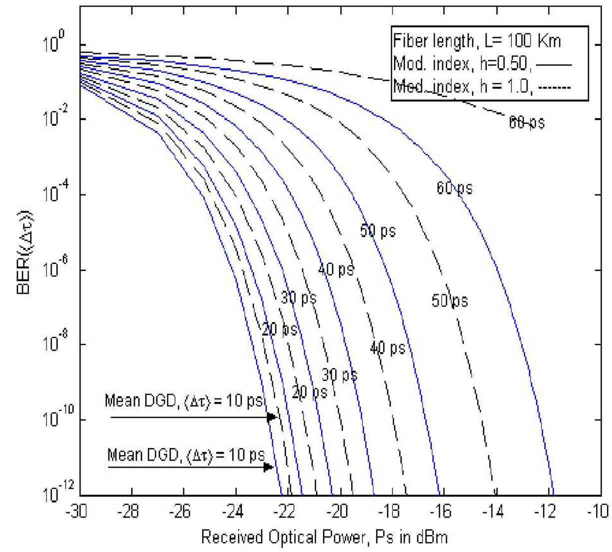

Fig.3: Plots of conditional BER versus received power, Ps for direct detection CPFSK system impaired by PMD

The amount of power penalty is found to be approximately 0.50 dB, 1.45 dB, 2.70 dB and 4.50 dB for mean DGD of 10 ps, 20 ps, 30 ps and 40 ps respectively when the modulation index is 0.50. It is further observed that the penalty is much higher at higher modulation index such as h=1 compared to h=0.50 i.e., the penalty is approximately 5.25 dB when mean DGD is 40 ps and h=1.0.

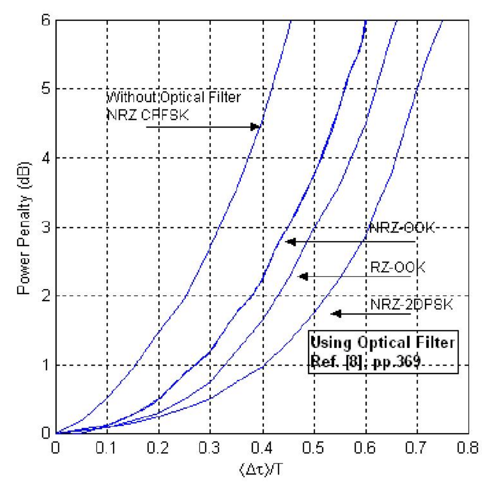

Fig.4: PMD-induced power penalty as a function of DGD/ bit duration ratio ($<\Delta\tau>/T$) for NRZ- and RZ-OOK, NRZ-2DPSK and NRZ-CPFSK system.

The penalty due to PMD suffered by the direct detection CPFSK system without post detection low pass filter at BER 10^{-9} is plotted in Fig. 4 as a function of DGD normalized by bit rate. The plots of penalty due to other modulation scheme as reported in [8] are also shown for comparison. It is found that penalty suffered by direct detection CPFSK is higher than that of OOK and DPSK system. For example, the penalty is 1.50 dB and 2.75. dB corresponding to DGD/bit duration ratio ($\Delta\tau/T$) are 0.20 and 0.30 respectively for direct detection CPFSK system The corresponding penalty values are 0.50 dB, 1.35 dB and 0.30 dB,0.80 dB. for NRZ-OOK and NRZ-DPSK respectively. The higher penalty in direct detection CPFSK is due to absence of low pass filtering and can be further reduced by using a post detection low pass filter.

E. Conclusion

An analytical approach is presented to evaluate the impact of PMD on the bit error rate performance of direct detection CPFSK system. The results show that the penalty due to PMD suffered by the CPFSK system is significant at higher modulation index and higher value of the DGD between two-polarization modes.

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