Performance of Lincoded Optical Heterodyne CPFSK Transmission System Affected by Polarization Mode Dispersion in a Single Mode Fiber

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***Abstract*- A theoretical analysis for evaluating the performance of a lincoded continuous phase optical frequency shift keying (CPFSK) optical transmission system with delay line demodulation receiver is presented. It accounts for the combined effects of signal phase distortion due to polarization mode dispersion (PMD) and group velocity dispersion (GVD) in a single mode fiber. The analysis is carried out for three different linecoding schemes, i.e., alternate mark inversion, order-1 and delay modulation coding, to investigate the efficacy of the linecoding in counteracting the effect of PMD in CPFSK optical transmission system. The average bit error rate (BER) performance results are evaluated with line codes and compared with NRZ-CPFSK system at a bit rate of 10 Gb/s considering Maxwellian probability distribution for the mean differential group delay (DGD) due to PMD. The results show that the penalty improvement of line coded systems is within 0.5 dB to 1.95 dB with respect to NRZ data at a BER of 10^{-9}.**

I. INTRODUCTION

The use of heterodyne or homodyne CPFSK optical communication systems offer the promise of significant improvement in receiver sensitivity compared with intensity modulation direct detection systems [1]. However, polarization mode dispersion (PMD) is considered to be one of the main obstacles for deploying high-speed optical communication systems operating at 10 Gb/s per channel or higher. A single mode fiber is actually a two mode fiber, because it can support two orthogonally polarized eigenmodes. Due to imperfection in fiber manufacture and deployment, the propagation constants of the two modes in a single mode fiber are generally different and mode coupling occurs as light propagates through a fiber. The different propagation speed results in PMD that leads to bit pattern corruption, higher bandwidth requirement, and eventually contributes to BER deterioration, performance variation or system fading even at moderate bit rates [2-3].

With the ever increasing bit rate in optical transmission systems, much effort has been spent on understanding PMD impairments and developing PMD mitigation techniques [4-8]. Due to the stochastic nature of PMD and the fact that all the orders of PMD are bound together, it is not trivial to evaluate PMD penalties and compensate for PMD impairments. PMD may be mitigated in three ways, i.e., passive technique [9], electronic compensation [10] and optical compensation [11-12]. Passive PMD mitigation techniques do not require any dynamically adjusted components and are bit rate independent. It includes use of more PMD robust line coded signal and soliton transmission, allocating more margin to PMD distortions, forward error correction (FEC) coding, channel switching and polarization scrambling. For DWDM systems, *Sarkimukka et al* [13] proposed a system by moving traffic from PMD-impaired channels onto spare channels that are not experiencing PMD degradation. It has been shown that with this approach five times as much as PMD can be tolerated compared to the suggested ITU limit (G.691).

In this paper, several linecoding schemes are applied to optical CPFSK transmission system with a delay line demodulation receiver in presence of PMD and GVD in a single mode fiber. We determine the average BER for different mean values of DGD, considering the Maxwellian probability density function for the DGD. The effectiveness of linecoded systems are evaluated in terms of reduction in power penalty due to PMD.

II. SYSTEM MODEL

The model of a heterodyne optical CPFSK system with delay line demodulation receiver is shown in Fig.1. At the transmitter side, the random NRZ data process representing the information signal is passed through a linecoder to shape the information signal spectrum. Three different line coding techniques, i.e., DM, AMI and order-1 coding are considered in this work. The linecoded data is used to modulate the DBF laser to generate the CPFSK signal.

The received optical signal is combined with the optical signal output from the local oscillator and the two signals are detected by a photo-detector and feed to an IF filter. The output of the filter is then demodulated using a delay line discriminator. The output of the discriminator is then filtered by a low-pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero than a binary ‘1’ is detected, otherwise a binary ‘0’ is detected.

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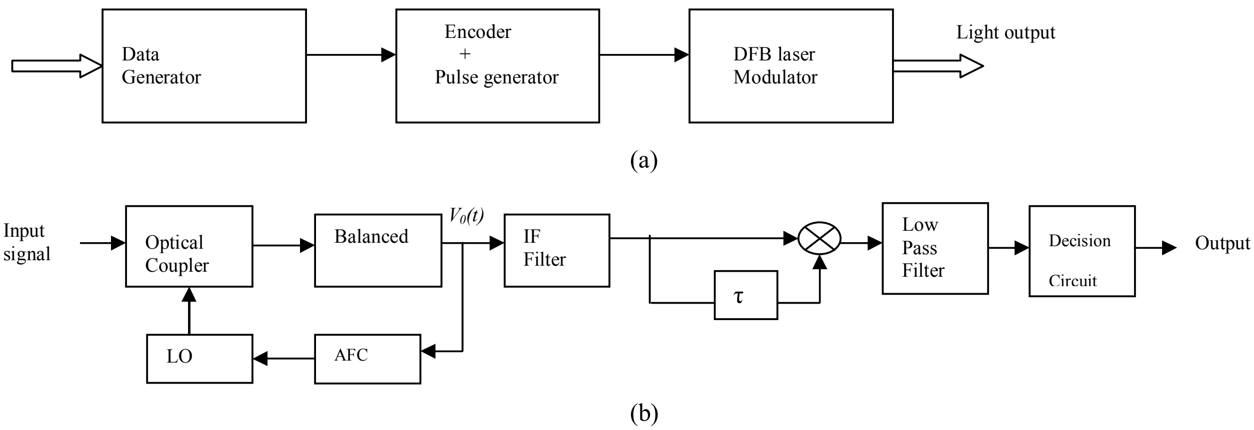

Fig. 1: Model of a heterodyne CPFSK system (a) transmitter, (b) delay line demodulation receiver

III. THEORETICAL ANALYSIS

The complex electric field at the output of the continuous phase FSK (CPFSK) transmitter and input to the fiber is represented as

$$E_1(t) = \sqrt{2P_T}\ \exp[j2\pi f_c t + j\phi_s(t)][c_1.e_1] \qquad (1)$$

$$E_2(t) = \sqrt{2P_T}\ \exp[j2\pi f_c t + j\phi_s(t)][c_2.e_2] \qquad (2)$$

where, f_c is the carrier frequency, P_T is the transmitted optical power and the CPFSK modulating phase $\phi_s(t)$ is given by

$$\phi_s(t) = 2\pi\,\Delta f \int_{-\infty}^{t} I(t)dt \quad + \phi_n(t) \qquad (3)$$

where, $I(t)$ is the modulating signal containing the bit sequence, Δf , is the peak frequency deviation and $\phi_n(t)$ is the phase noise of the transmitting laser. Here, c_1, c_2 represent unit vectors and e_1, e_2 represent the two principal states of polarization (PSPs) respectively.

The electric field output of the fiber is then given by

$$E_{01}(t) = \sqrt{2P_T}\ \exp[j2\pi f_c t + j\phi_s(t)][c_1.e_1] \otimes h_1(t) \qquad (4)$$

$$E_{02}(t) = \sqrt{2P_T}\ \exp[j2\pi f_c t + j\phi_s(t)][c_2.e_2] \otimes h_2(t) \qquad (5)$$

where $\otimes$ denotes convolution, $h_1(t)$ and $h_2(t)$ are the inverse Fourier transform of the low pass equivalent transfer function of a non-dispersion shifted lossless fiber $H_1(f)$ and $H_2(f)$ respectively, which include the effect of PMD and group velocity dispersion and are given by

$$H_1(f) = \sqrt{\alpha}\exp[\ j2\pi f(-\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \qquad (6)$$

$$H_2(f) = \sqrt{1-\alpha}\exp[\ j2\pi f(\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \qquad (7)$$

where α is the PMD power splitting ratio, $\Delta\tau$ represents the DGD between the two PSPs and γ is chromatic dispersion index. Here, we assume that there is a negligible amount of polarization dependent loss.

Now, assuming linear phase approximation, the output electric fields for two polarization states can be given as

$$E_{01}(t) = \sqrt{2P_s}\ \exp[j2\pi f_c t + j\phi_{01}(t)][c_1.e_1] \qquad (8)$$

$$E_{02}(t) = \sqrt{2P_s}\ \exp[j2\pi f_c t + j\phi_{02}(t)][c_2.e_2] \qquad (9)$$

where, $$\phi_{01}(t) = 2\pi\,\Delta f \int_{-\infty}^{t} \sum_k a_k \operatorname{Re}[\,p(t-kT)\otimes h_1(t)]dt \qquad (10)$$

$$\phi_{02}(t) = 2\pi\,\Delta f \int_{-\infty}^{t} \sum_k a_k \operatorname{Re}[\,p(t-kT)\otimes h_2(t)]dt \qquad (11)$$

The signal at the output of the IF filter is given by

$$i_0(t) = R_d\sqrt{P_s P_{Lo}}\{\exp[j2\pi f_{IF}t + j\phi'_{01}(t)] + \exp[j2\pi f_{IF}t + j\phi'_{02}(t)]\} + i_n(t) \qquad (12)$$

where $i_n(t)$ represents the total noise current consisting of shot noise and receiver thermal noise, R_d is the responsivity of the photodetector, P_s is the output power at the fiber end, P_{Lo} is the local oscillator power, f_{IF} is the IF frequency. The phases $\phi'_{01}(t)$ and $\phi'_{02}(t)$ are given by

$$\phi'_{01}(t) = 2\pi\ \Delta f \int_{-\infty}^{t} \sum_k a_k g_1(t-kT)dt \qquad (13)$$

$$\phi'_{02}(t) = 2\pi\ \Delta f \int_{-\infty}^{t} \sum_k a_k g_2(t-kT)dt \qquad (14)$$

where $g_1(t) = \operatorname{Re}[p(t)\otimes h_1(t)\otimes h_{IF}(t)]$
$g_2(t) = \operatorname{Re}[p(t)\otimes h_2(t)\otimes h_{IF}(t)]$

where $h_{IF}(t)$ is the impulse response of the IF filter. The IF signal current is then given by,

$$i_0(t) = 2R_d\sqrt{P_s P_{Lo}}\ \cos\phi''_0(t)\cos[2\pi f_{IF}t + \phi'_0(t)] + i_n(t) \qquad (15)$$

where , $\phi_0'(t) = \frac{\phi_{01}' + \phi_{02}'}{2}$ and $\phi_0''(t) = \frac{\phi_{01}' - \phi_{02}'}{2}$

The output of the IF filter can be expressed as

$$v_0(t) = 2R_d \sqrt{P_s P_{Lo}}\ a(t)\cos[2\pi f_{IF} t + \phi_0'(t)] + n(t) \qquad (16)$$

where, $a(t) = \cos\phi_0''(t)$, which is the amplitude fluctuations due to PMD and GVD, $n(t)$ is the receiver shot noise with variance $\sigma_n^2 = 2e(Ps + P_{Lo})B_{IF}$, B_{IF} being the IF filter bandwidth.

Following the delay line demodulation (delay time τ) and low pass filtering (wide enough to pass the signal undistorted), the low pass filter (LPF) output is given by

$$v_{out}(t) = A(t)\cos[\Delta\phi_0(t,\tau)] \qquad (17)$$

where $A(t) = 2R_d\sqrt{P_s P_{Lo}}\cos\phi_0''(t)$; $A(t)$ is the amplitude and the differential output phase of $v_{out}(t)$ of the LPF can be expressed as

$$\Delta\phi(t,\tau) = 2\pi f_{IF}\tau + \Delta\phi_0'(t,\tau) + \Delta\theta_n(t,\tau) \qquad (18)$$

A. CPFSK with NRZ data

For a CPFSK system with NRZ data, the driving signal of the transmitted laser is given by

$$I(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT) \qquad (19)$$

with $a_k = \pm 1$ is the kth bit of information bit, $p(t)$ is the elementary pulse shape of the individual data bit with the of duration T seconds.

In delay line demodulation receiver the data decisions are based on the polarity of $v_{out}(t)$. Assuming a 'mark' is transmitted (say, $a_0 = 1$) and under ideal CPFSK demodulation condition, $2\pi f_{IF}\tau = (2n+1)\frac{\pi}{2}$; n is an integer and $2\pi\,\Delta f\tau = \frac{\pi}{2}$ for NRZ data. Now, applying the above conditions, the phase of $v_{out}(t)$ at the sampling instant t_0 can be written as

$$\begin{aligned}\Delta\phi_0(t_0,\tau) = 2\pi f_{IF}\tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2}q(t_0) + \\ \frac{\pi}{2}\sum_{k\neq 0} q(t_0 - kT) + \Delta\theta_n(t_0,\tau)\end{aligned} \qquad (20)$$

Where $q(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_0(t)dt$ and $g_0(t) = \frac{1}{2}[g_1(t) + g_2(t)]$

Note that the first two terms in (20) provide the expected phase change during the demodulation interval τ corresponding to the ideal situation, the next three terms, in fact, represent the undesired contribution to the phase due to PMD and GVD and the last term, $\Delta\theta_n(t_0,\tau)$ represents the phase distortion due to receiver noise.

Denoting the phase distortion of the signal only due to PMD and GVD as η

$$\eta = -\frac{\pi}{2} + \frac{\pi}{2}q(t_0) + \frac{\pi}{2}\sum_{k\neq 0} a_k q(t_0 - kT) \qquad (21)$$

$$= \overline{\Delta}\alpha(t,\tau) + \xi \qquad (22)$$

where $\overline{\Delta}\alpha(t,\tau)$ is the mean output phase error and ξ is random output phase fluctuation due to ISI for random bit pattern caused by PMD and GVD.

We define the IF SNR as $\rho = V_0^2 / 2\sigma_n^2$. For a given IF SNR, the conditional bit error probability, $P(e\,|\,\xi)$ conditioned on a given value of ξ and differential group delay $\Delta\tau$, can be obtained following Ref. [15].

For a given value of instantaneous differential group delay (DGD), $\Delta\tau$ the conditional BER can be expressed as

$$BER(\Delta\tau) = \int_{-\infty}^{\infty} P(e\,|\,\xi)\,P_\xi(\xi)\,d\xi \qquad (23)$$

where, $P_\xi(\xi)$ is the probability density function (pdf) of ξ and can be found by following Ref. [14].

The statistical properties of PMD have been experimentally and theoretically studied and found that the evolution of DGD at particular frequency over time yields a Maxwellian probability density function given by [16]

$$P_{\Delta\tau}(\Delta\tau) = \frac{32\,\Delta\tau^2}{\pi^2\,\langle\Delta\tau\rangle^3}\exp\left(-\frac{4\,\Delta\tau^2}{\pi\,\langle\Delta\tau\rangle^2}\right), \quad \Delta\tau > 0 \qquad (24)$$

where $\langle\Delta\tau\rangle$ is the quadratic mean of instantaneous DGD, $\Delta\tau$

The average bit error probability can now be evaluated as

$$BER\left(\langle\Delta\tau\rangle\right) = \int_{-\infty}^{\infty} BER(\Delta\tau)\,P_{\Delta\tau}(\Delta\tau)\,d(\Delta\tau) \qquad (25)$$

B. CPFSK with linecoding

Our main objective is to study the relative effectiveness of the above linecodes in overcoming the pulse dispersion performance degradation caused by PMD. The linecoder shown in Fig. 1(a) can be viewed as a linear time-invariant filter that acts on the input data signal of (19) to generate an output sequence of data given by

$$I'(t) = \sum_{k=-\infty}^{\infty} b_k r(t - kT) \qquad (26)$$

where b_k's are independent and identically distributed (iid) random variable taking values ±1 and $r(t) = p(t) \otimes c(t)$. Fourier transform of $r(t)$ can be written as

$$R(f) = C(f).P(f) \qquad (27)$$

where,

$$C(f) = \sum_{k=0}^{K-1} c_k \exp(-j2\pi kfT) \qquad (28)$$

is the transfer function of the encoder and where $c(t) = F^{-1}[G(f)]$ is pulse shaping due to linecoding. The sequence of symbols $\{b_k\}$ is generated to avoid error propagation by a precoder defined by

$$a_k = \sum_{i=0}^{K-1} c_i b_{k-i} \bmod(2) \quad (29)$$

The K code coefficients $c_0, c_1, c_2 ....c_{K-1}$ are usually integers and can be properly chosen to generate a specific linecoding scheme.

Therefore, it is possible to apply the same procedure for average BER performance evaluation of CPFSK system with NRZ data in Section A to the present situation with lincoded data by merely replacing $g_1(t)$ and $g_2(t)$ in the previous case by

$$g_1(t) = \mathrm{Re}[p(t) \otimes c(t) \otimes h_1(t) \otimes h_{IF}(t)] \quad (30)$$

$$g_2(t) = \mathrm{Re}[p(t) \otimes c(t) \otimes h_2(t) \otimes h_{IF}(t)] \quad (31)$$

IV. RESULTS AND DISCUSSION

To assess the performance of the lincodes discussed in the preceding sections, a 10 Gb/s scenario was used with delay line demodulation receiver for several values of mean DGD. We assume the worst-case value of power splitting between the two principal states of polarization's, i.e., $\alpha = 0.50$ in (6) and (7). The plots of average BER versus optical received power for AMI, order-1 and DM linecoding are shown in Fig. 2, Fig. 3 and Fig.4 respectively at modulation index of 0.50 and 1.00. The results show that the BER performance is more affected at higher values of mean DGD and higher modulation index.

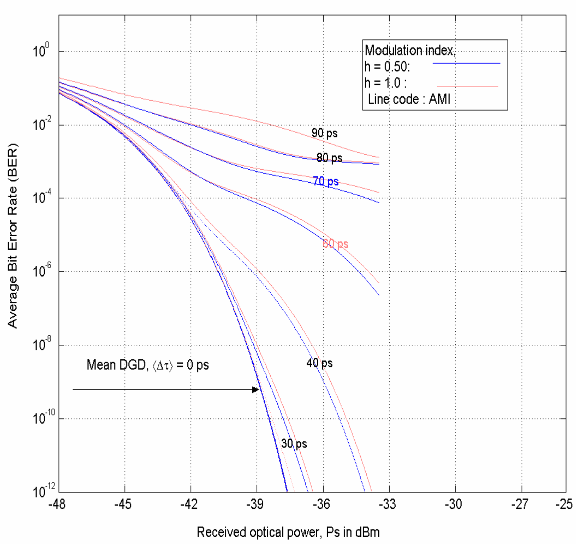

Fig. 2: Plot of average BER versus received power, Ps with Alternate mark inversion (AMI) linecoding (Modulation index h=0.50 and h= 1.0)

It is found from Fig. 2 and 3 that the amount of power penalty due to PMD suffered by CPFSK transmission system up to 30 ps mean DGD is relatively small. It is approximately 0.85dB and 0.30 dB for AMI-CPFSK and order-1-CPFSK respectively at BER 10^{-9}. For delay demodulation (Fig.4) linecoding, it is about 1.5 dB at the same BER and 30 ps mean DGD.

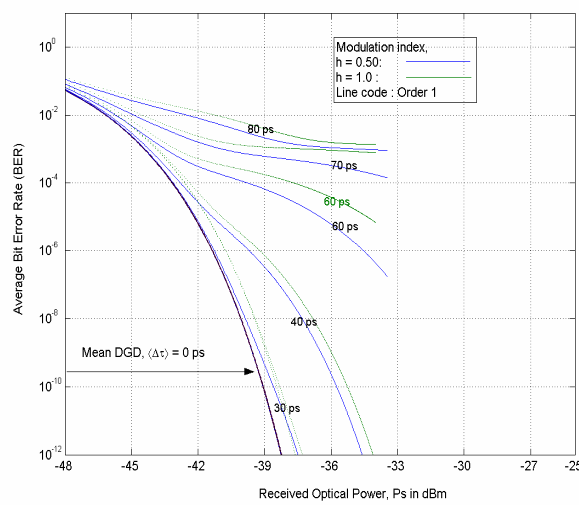

Fig. 3: Plot of average BER versus received power, Ps with Order-1 linecoding (Modulation index h=0.50 and h= 1.0)

For small power penalties PMD follows the following expression [17]

$$\varepsilon\ (in\ dB) = A\left(\frac{\langle\Delta\tau\rangle}{MT}\right)^2 \alpha(1-\alpha) \quad (32)$$

Where, MT is the symbol duration A is a dimensionless parameter determined by the pulse shape. In our case, M=1 and the value of A is about 39, 22, 19 and 16 corresponding to NRZ, DM, AMI and order-1 codes respectively.

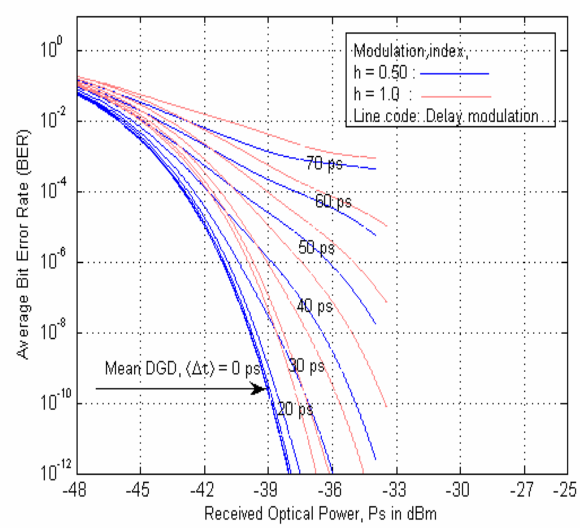

Fig. 4: Plot of average BER versus received power, Ps with DM linecoding (Modulation index h=0.50 and h= 1.0)

It is also noticed that BER floors occur at about 10^{-4} (at DGD 70 ps), 3.5×10^{-5} (at DGD 60 ps) and 5×10^{-5} (at DGD 60 ps) for AMI, order-1 and delay modulation linecoding respectively. Above these values of mean DGD, the BER can not be lowered by further increasing the signal power. The assumptions used in computing the BER performance results are shown in Table 1.

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Fig. 5 shows the 10^{-9} BER power penalty of NRZ- , AMI- , order-1- and DM-CPFSK, versus the ratio of mean DGD to bit duration $\langle \Delta\tau \rangle / T$ at a modulation index of 0.50. The performance limitations of an optical heterodyne NRZ-CPFSK transmission system affected by PMD is reported in Ref. [14]. As seen in Fig.5, for weak PMD the effect of linecoding more pronounced and order-1 linecoding is more superior to AMI and DM. For example, the power penalty improvements are about 0.5 dB, 1.15 dB and 1.95 dB corresponding to DM, AMI and order-1 respectively at mean DGD of 30 ps with respect to NRZ-CPFSK.

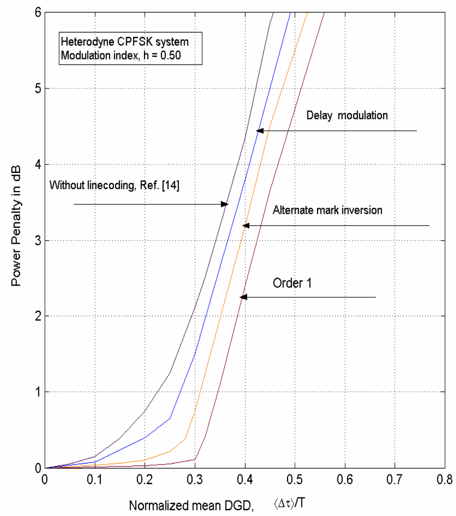

Fig.5: PMD-induced power penalty as a function of DGD/bit duration ($(< \Delta\tau > / T$) for NRZ, DM, AMI and Order-1 linecoding of a CPFSK system (modulation index =0.50)

Table 1: Assumptions

Parameter	Values	unit
Temperature (T)	300	K
Bandwidth (B)	10	GHz
Receiver load (R_L)	50	Ohm
Receiver responsivity (R_d)	1	A/W
Wavelength (λ)	1550	nm
Operating frequency (f)	$1.93355\text{x}10^{14}$	Hz
Scaling factor (β)	0.50	Unitless
Dispersion index (γ)	15	ps/nm.km
Power splitting ratio (α)	0.50	Unitless
Fiber length (L)	100	km

From Fig.5, it is also observed that the power penalty improvements above 30 ps mean DGD is almost constant for all linecodes and the amounts are about 0.30 dB, 0.80 dB and 1.65 dB corresponding to DM, AMI and order-1 respectively. This is to be expected, because in NRZ, the symbol duration is twice as long as for three linecoding schemes. As these linecodes use shorter pulse duration and exhibit much greater PMD tolerance than their NRZ counterparts. It is also observed that order-1 linecoding gives nearly twice the maximum allowable value of $\langle \Delta\tau \rangle / T$ than NRZ data.

IV. COCLUSION

In this paper, an analytical technique without and with linecoding is provided for the performance evaluation of a heterodyne CPFSK system impaired by PMD. The effectiveness of several linecoding schemes, ie, AMI, order-1 and DM, in alleviating the performance degradation due to the PMD are quantitatively assessed. We found that among the linecoding schemes considered order-1 coding exhibits superior performance in the presence of GVD and PMD over that of AMI and DM coding due its compact spectrum.

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