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Impact of Polarization Mode Dispersion on CPFSK Transmission Systems using MZI Based Direct Detection Receiver

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Summary

An analytical approach is presented to determine the impact of signal phase distortion due to polarization mode dispersion (PMD) in a single mode fiber (SMF) on the bit error rate (BER) performance of an optical continuous phase FSK (CPFSK) transmission system with MZI based direct detection receiver. The probability density function (pdf) of the random phase fluctuation due to PMD and group velocity dispersion (GVD) at the output of the receiver is determined analytically. Based on the pdf of the random phase fluctuation the BER performance results are evaluated for different values of the mean differential group delay (DGD) and also by considering Maxwellian distribution of it at a bit rate of 10 Gbit/s. The computed results show that the direct detection CPFSK system suffers a significant amount of power penalty and occurrence of BER floor due the effect of PMD. At a modulation index of 0.5, the amount of power penalty at a BER of 10^{-9} is found to be 0.5 dB, 1.10 dB, 2.10 dB and 3.50 dB when the mean DGD is 20 ps, 30 ps, 40 ps and 50 ps respectively for a fiber length of 100 km. At modulation index of 1.0, the penalty is approximately 4.10 dB for a mean DGD of 40 ps. Furthermore, while considering Maxwellian distribution for mean DGD, there occur BER floors above 10^{-9}, which cannot be lowered by further increasing the signal power. It is noticed that BER floors occur at about 10^{-4} and 10^{-8} for modulation index of 0.50 and 1.0 respectively to a corresponding mean DGD of 90 ps. The effect of PMD is found to be more detrimental at higher modulation index and bit rate.

1 Introduction

Optical continuous-phase FSK (CPFSK) is an attractive modulation format as it allows generation of compact spectra that allows for receiver detection by properly selecting the modulation index [1]. However, recent progress in long-haul optical repeater systems indicates that the PMD influence has become one of the main degradation factors in high-speed and wideband systems. The PMD influence becomes a serious problem when one bit time or the inverse of occupied bandwidth is comparable to the PMD value in both intensity modulation-direct detection and coherent system. The PMD induces the differential group delay (DGD) between two principal polarization axes of the transmission media and distorts the signal waveform that results inter-symbol interference (ISI) and higher bandwidth requirement, and eventually contributes to BER deterioration, performance variation or system fading even at moderate bit rate [2].

The effect of PMD has been the focus of research in fiber communications for more than a decade and the developed ideas are briefly described in a series of publication [3-10]. The effect for PMD on the error probability is theoretically assessed at a bit rate of 10 Gbit/s for an IM-DD system and the BER in a IM/DD has been shown to strongly depend on the state of polarization and DGD [3]. The simulation results on the effects of PMD on an amplified IM-DD system is reported in [4] as a function of the DGD. The effect of PMD on the bit error rate performance of heterodyne FSK system is also reported [7]. Recently, the performance of optical DPSK system with direct detection receiver is reported in presence of PMD [8]. Several attempts have been reported to compensate the effect of PMD in electronic as well as in optical domain [9, 10]. Although there are many approaches reported to minimize the effect of PMD, there is no analytical development to evaluate the impact of PMD on the BER performance of an optical transmission system.

In this paper, we provide an analytical approach to evaluate the BER performance limitations of an optical direct detection CPFSK system impaired by PMD. This method is based on the linear approximation of the output phase of a linearly filtered angle-modulated signal such as the CPFSK signal. The probability density function (pdf) of the random phase fluctuation due to PMD and group velocity dispersion (GVD) at the output of the receiver is determined analytically. Based on the pdf of the random phase fluctuation, a conditional BER expression is derived and BER performance is evaluated. The temporal behaviors of the fiber PMD are of statistical in nature due to randomness of the birefringence variations along the fiber structure. Therefore to assess accurately the optical transmission impairment due to PMD, we determine the average BER and power penalty for different mean values of DGD, considering Maxwellian prob-

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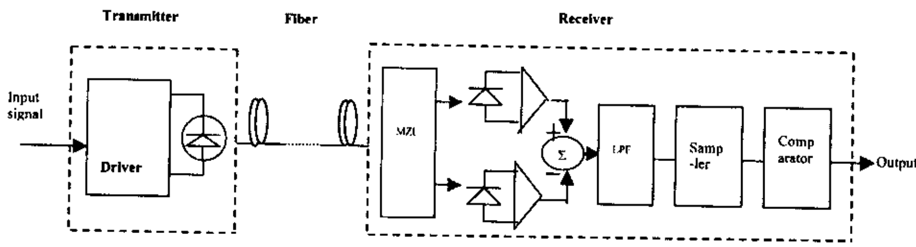

Fig. 1: Block diagram of an optical CPFSK transmission system with MZI based direct detection receiver

ability density function for the DGD, at the receiver output in the presence of receiver noise. The average bit error rate performance and power penalty suffered by the system due to PMD at a BER of 10^{-9} are evaluated at a bit rate of 10 Gbit/s.

2 System model

The block diagram of an optical CPFSK transmission system with direct detection model is shown in Fig.1. Figure 2 shows the low pass equivalent CPFSK system model considering PMD and chromatic dispersion. The signal components in the two output PSPs propagate independently through the entire system from the modulator to the balanced receiver. In the transmitter, non-return to zero (NRZ) data at 10 Gbit/s is used to directly modulate a laser to generate the CPFSK signal that is transmitted through a single mode fiber. At the receiving end, MZI based direct detection receiver detects the received optical signal. The fields corresponding to the two PSPs are separately detected and the resulting photocurrents are summed. We decompose the entire system into a pair of subsystems, each corresponding to one PSP. We sum the outputs of the two subsystems to obtain sampled decision variables $i_m(t) = i_{m1}(t) + i_{m2}(t)$. The output photocurrent is filtered by a low pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero a binary '1' is detected, otherwise a binary '0' is detected.

3 Theoritical analysis

The complex electric field at the output of the CPFSK transmitter and input to the fiber is represented as:

$$E_1(t) = \sqrt{2P_T}\exp[j\{2\pi f_c t + \phi_s(t)\}][c_1.e_1] \quad (1)$$

$$E_2(t) = \sqrt{2P_T}\exp[j\{2\pi f_c t + \phi_s(t)\}][c_2.e_2] \quad (2)$$

where f_c is the carrier frequency, P_T is the transmitted optical power and the CPFSK modulating phase $\phi_s(t)$ is given by

$$\phi_s(t) = 2\pi\,\Delta f \int_{-\infty}^{t} \sum_{k=-\infty}^{\infty} a_k p(t-kT)dt + \phi_n(t) \quad (3)$$

where a_k is the random NRZ bit, Δf is the peak frequency deviation, $p(t)$ is the elementary pulse shape and $\phi_n(t)$ is the phase noise of the transmitting laser. Here c_1, c_2 represent unit vectors and e_1, e_2 represent the two principal states of polarization respectively. The electric field at the output of the fiber is then given by

$$E_{01}(t) = \sqrt{2P_T}\exp[j\{2\pi f_c t + \phi_s(t)\}][c_1.e_1] \otimes h_1(t) \quad (4)$$

$$E_{02}(t) = \sqrt{2P_T}\exp[j\{2\pi f_c t + \phi_s(t)\}][c_2.e_2] \otimes h_2(t) \quad (5)$$

where $\otimes$ denotes convolution, $h_1(t)$ and $h_2(t)$ are the inverse Fourier transform of the low pass equivalent transfer function of a non-dispersion shifted lossless fiber $H_1(f)$ and $H_2(f)$ respectively, which include the effect of polarization mode dispersion (PMD) and group velocity dispersion (GVD) and given by [8]

$$H_1(f) = \sqrt{\alpha}\;\exp[j2\pi f(-\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \quad (6)$$

$$H_2(f) = \sqrt{1-\alpha}\;\exp[j2\pi f(\frac{\Delta\tau}{2}) - j\gamma(\pi f T)^2] \quad (7)$$

where $\Delta\tau$ represents the differential group delay (DGD) between the two principal states of polarization (PSP),

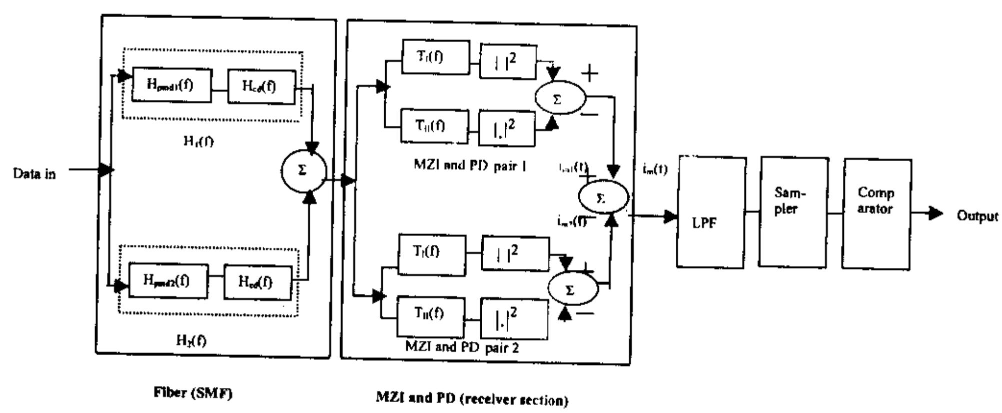

Fig. 2: Low pass equivalent direct detection CPFSK system model considering PMD and chromatic dispersion

α is the PMD power-splitting ration and γ is the chromatic dispersion index. Here we assume that there is negligible polarization-dependent loss.

Assuming linear phase approximation the output electric fields for the two polarization states can be written as

$$E_{01}(t) = \sqrt{2P_s}\exp[j\{2\pi f_c t + \phi_{s1}(t) + \theta_{n1}(t)\}][c_1 \cdot e_1] \quad (8)$$

$$E_{02}(t) = \sqrt{2P_s}\exp[j\{2\pi f_c t + \phi_{s2}(t) + \theta_{n2}(t)\}][c_2 \cdot e_2] \quad (9)$$

where

$$\phi_{s1}(t) = \mathrm{Re}[\phi_s(t) \otimes h_1(t)] \quad (10)$$

$$= 2\pi\,\Delta f \int_{-\infty}^{t} \sum_k a_k \,\mathrm{Re}[p(t-kT) \otimes h_1(t)]dt \quad (10\text{-}1)$$

$$\phi_{s2}(t) = \mathrm{Re}[\phi_s(t) \otimes h_2(t)] \quad (11)$$

$$= 2\pi\,\Delta f \int_{-\infty}^{t} \sum_k a_k \,\mathrm{Re}[p(t-kT) \otimes h_2(t)]dt\ . \quad (11\text{-}1)$$

In the FSK direct detection receiver with Mach-Zehnder interferometer (MZI), the MZI act as an optical discriminator and differentially detect the 'mark' and 'space' of the received FSK signal, which are then directly fed to a pair of photo-detectors. The transmittances of the two branches of the MZI are $T_I(f)$ and $T_{II}(f)$, and τ is the time delay between the two branches.

For a 'mark' transmission, the current at the output of the photo-detectors can be expressed as:

$$i_{m1}(t) = R_d P_s \cos[\,2\pi f_c \tau \quad + \Delta\phi_{s1}(t,\tau) + \Delta\theta_{n1}(t,\tau)][c_1 \cdot e_1] \quad (12)$$

$$i_{m2}(t) = R_d P_s \cos[\,2\pi f_c \tau + \Delta\phi_{s2}(t,\tau) + \Delta\theta_{n2}(t,\tau)][c_2 \cdot e_2] \quad (13)$$

where

$$\Delta\phi_{s1}(t,\tau) = \phi_{s1}(t) - \phi_{s1}(t-\tau) = 2\pi\,\Delta f \int_{t-\tau}^{t} \sum_k a_k g_1(t-kT)dt \quad (14)$$

$$\Delta\phi_{s2}(t,\tau) = \phi_{s2}(t) - \phi_{s2}(t-\tau) = 2\pi\,\Delta f \int_{t-\tau}^{t} \sum_k a_k g_2(t-kT)dt \quad (15)$$

For a 'mark' transmission and ideal CPFSK demodulation condition (assuming NRZ data), we have $2\pi f_c \tau = (2n+1)\frac{\pi}{2}$, where n is an integer; and $2\pi\,\Delta f \tau = \frac{\pi}{2}$. Thus, the currents at the output of photodetector can be expressed as:

$$i_{m1}(t) = R_d P_s \cos[\,2\pi f_c \tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2} \sum_{k\neq 0} a_k q_1(t-kT) + \Delta\theta_{n1}(t,\tau)][c_1 \cdot e_1] \quad (16)$$

$$i_{m2}(t) = R_d P_s \cos[\,2\pi f_c \tau + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2} \sum_{k\neq 0} a_k q_2(t-kT) + \Delta\theta_{n2}(t,\tau)][c_2 \cdot e_2] \quad (17)$$

where, $q_1(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_1(t)dt$;

$$g_1(t) = \mathrm{Re}[p(t) \otimes h_1(t) \otimes h_{LP}(t)]$$

and $q_2(t) = \frac{1}{\tau}\int_{t-\tau}^{t} g_2(t)dt$;

$$g_2(t) = \mathrm{Re}[p(t) \otimes h_2(t) \otimes h_{LP}(t)]$$

where $h_{LP}(t)$ represents the impulse response of the electrical low pass filter, which is assumed to be fifth-order Bessel function type. The transfer function of the low pass filter is given by

$$H_{LP}(f) = \frac{945}{jF^3 + 15F^4 - 105jF^3 - 420F^2 + 945jF + 945} \quad (18)$$

where $F = 2.43f/B_e$ and B_e is the 3-dB cutoff frequency. The output phase distortion due to PMD and GVD is given by

$$\Delta\phi_1(t,\tau,\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_1(t-kT) = \overline{\Delta\varphi}_{s1}(t,\tau) + \xi_1(t,\tau) \quad (19)$$

$$\Delta\phi_2(t,\tau.\Delta\tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2}\sum_{k\neq 0} a_k q_2(t-kT) = \overline{\Delta\varphi}_{s2}(t,\tau) + \xi_2(t,\tau) \quad (20)$$

where $\overline{\Delta\varphi}_{s1}(t,\tau)$ and $\overline{\Delta\varphi}_{s2}(t,\tau)$ are mean output phase error; $\xi_1(t,\tau)$ and $\xi_2(t,\tau)$ are the random output phase fluctuation due to ISI bit pattern caused by PMD and GVD. The total signal current at the output of LPF corresponding to a 'mark' is given by

$$i_m(t) = R_d P_s x_1(t) + R_d P_s x_2(t) \quad (21)$$

where, $x_1(t)$ and $x_1(t)$ describe the phase noise induced by the interferometer intensity due to random phase fluctuation and can be expressed as:

$$x_1(t) = \cos[\Delta\varphi_1(t)] \quad (22)$$

$$x_2(t) = \cos[\Delta\varphi_2(t)] \quad (23)$$

where

$$\Delta\varphi_1(t) = \Delta\phi_1(t,\tau,\Delta\tau) + \Delta\varphi_n(t) \quad (24)$$

$$\Delta\varphi_2(t) = \Delta\phi_2(t,\tau,\Delta\tau) + \Delta\varphi_n(t)\ . \quad (25)$$

The low-pass filter output for 'mark' transmission, at a sampling time t is

$$i_m(t) = 2R_d P_S\,[\cos(\delta\varphi)\cos(\Delta\varphi)] + n(t) \tag{26}$$

where

$$\delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) - \Delta\phi_2(t,\tau,\Delta\tau)}{2} \tag{27}$$

and
$$\Delta\varphi = \frac{\Delta\phi_1(t,\tau,\Delta\tau) + \Delta\phi_2(t,\tau,\Delta\tau)}{2} . \tag{28}$$

If we consider the mean phase distortion only then the conditional bit error rate can be expressed as:

$$P(e\,|\,\overline{\delta\varphi},\, \overline{\Delta\varphi}) = \frac{1}{2}\,erfc\left(Q\big/\sqrt{2}\right) \tag{29}$$

where

$$Q = \frac{2R_d P_S\,[\cos(\overline{\delta\varphi})\cos(\overline{\Delta\varphi})]}{\sigma_m} . \tag{30}$$

The contribution of the term (random phase fluctuation) $\xi_1(t,\tau)$ of (19) and $\xi_2(t,\tau)$ of (20) can be evaluated by finding the probability density function (pdf). Assuming $\{a_k\}$ are independent and identically distributed (IID) random variables, the characteristic function of ξ_1 and ξ_2 is given by

$$\Phi_{\Delta\phi_1}(j\xi_1) = \prod_{i=1}^{\infty}\cos[\xi_1 q_{1i}(t)] = \sum_{i=1}^{\infty}\frac{(j\xi_1)^{2i}}{(2i)!}M'_{2i} \tag{31}$$

$$\Phi_{\Delta\phi_2}(j\xi_2) = \prod_{i=1}^{\infty}\cos[\xi_2 q_{2i}(t)] = \sum_{i=1}^{\infty}\frac{(j\xi_2)^{2i}}{(2i)!}M''_{2i} \tag{32}$$

where $q_{1i}(t) = \left|q_1(t-iT)\right|$, $q_{2i}(t) = \left|q_2(t-iT)\right|$, M'_{2i} and M''_{2i} are the even order moments of the characteristic function of random variable ξ_1 and ξ_2 respectively. By using the above two equations the pdf $P_{\delta\varphi}(\delta\varphi)$ of $\delta\varphi$ and the pdf $P_{\Delta\varphi}(\Delta\varphi)$ of $\Delta\varphi$ can be evaluated following [7].

For a given value of differential group delay (DGD) $\Delta\tau$, the conditional BER can be expressed as:

$$BER(\Delta\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(e\,|\,\overline{\delta\varphi},\overline{\Delta\varphi})\, P_{\delta\varphi}(\delta\varphi)\,P_{\Delta\varphi}(\Delta\varphi)\, d(\delta\varphi)\, d(\Delta\varphi) . \tag{33}$$

PMD is a stochastic process. The random configuration of birefringence, which causes PMD, depends on the stress induced by spooling, cabling, temperature changes and any other environmental factor that may cause the core of the fiber to deviate from being perfectly cylindrical. The statistical properties of PMD have been experimentally and theoretically studied and found that the evolution of DGD at particular frequency over time yields a Maxwellian probability density function given by

$$P_{\Delta\tau}(\Delta\tau) = \frac{32\Delta\tau^2}{\pi^2\langle\Delta\tau\rangle^3}\exp\left(-\frac{4\Delta\tau^2}{\pi\langle\Delta\tau\rangle^2}\right), \quad \Delta\tau > 0 \tag{34}$$

where $\langle\Delta\tau\rangle$ is the quadratic mean of instantaneous DGD, $\Delta\tau$. The average bit error probability can now be evaluated as:

$$BER = \int_{-\infty}^{\infty} BER(\Delta\tau)\, P_{\Delta\tau}(\Delta\tau)\, d(\Delta\tau) \tag{35}$$

4 Results and discussion

Following the analytical approach, the bit error rate performance results for direct detection CPFSK receiver with post detection low pass filter are evaluated at a bit rate of 10 Gbit/s with several values of mean DGD and fiber lengths. The plots of conditional BER versus received optical power Ps are shown in Fig. 3 for mean DGD of 10 ps, 20 ps, 30 ps, 40 ps, 50 ps and 60 ps with fiber length of 100 km and modulation index of 0.50 and plots for modulation index 1.0 are shown in Fig.4.

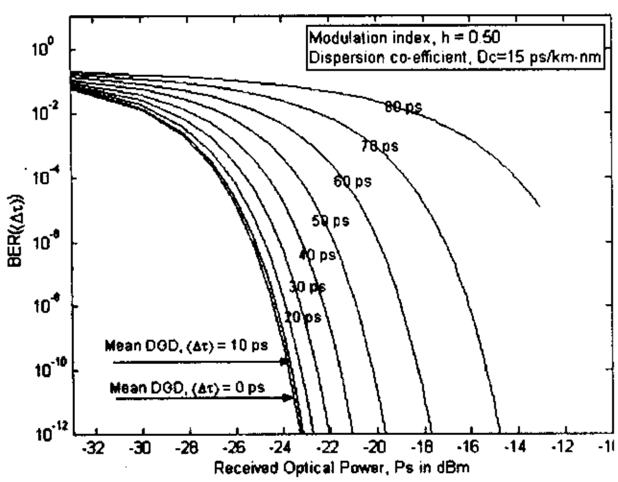

Fig. 3: Conditional BER versus received power, P_s for direct detection CPFSK system impaired by PMD (modulation index, h=0.50)

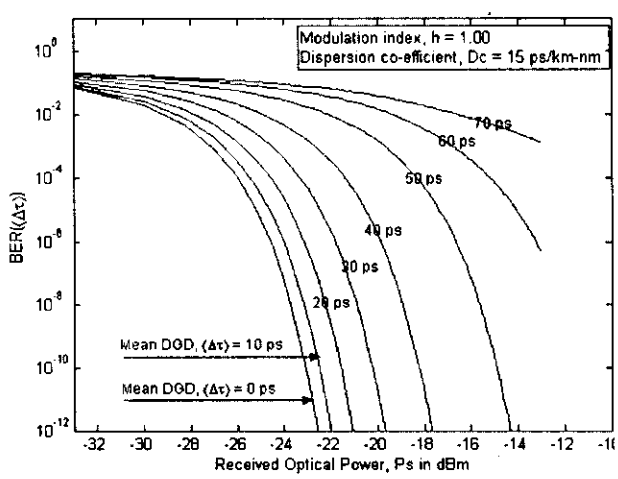

Fig. 4: Conditional BER versus received power, P_s for direct detection CPFSK system impaired by PMD (modulation index, h=1.0)

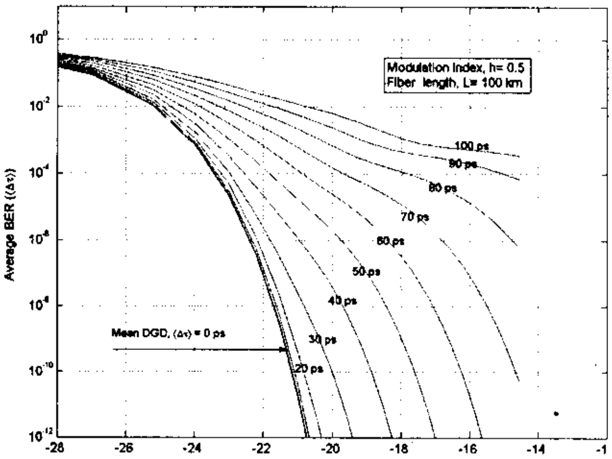

Fig. 5: Plots of average BER versus received power, Ps for direct tetection CPFSK system impaired by PMD (modulation index h = 0.50)

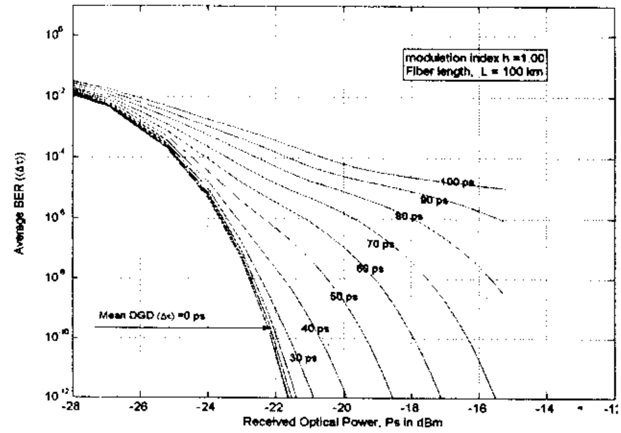

Fig. 5: Plots of average BER versus received power, Ps for direct tetection CPFSK system impaired by PMD (modulation index h = 1.0)

From Fig. 4 and Fig. 5, it is found that the amoun' of power penalty is found to be approximately 0.50 dB, 1.10 dB, 2.10 dB and 3.50 dB for mean DGD of 20 ps, 30 ps, 40 ps and 50 ps respectively when the modulation index is 0.50. It is further observed that the penalty is much higher at higher modulation index such as h=1 compared to h=0.50 *i.e.*, the penalty is approximately 4.10 dB when mean DGD is 40 ps and h=1.0.

The plots of average BER versus received power, Ps are shown in Fig. 5 and Fig. 6 at modulation index of 0.50 and 1.0 respectively for different values of mean DGD at a bit rate of 10 Gbit/s. The results show that the BER performance is more affected at higher values of mean DGD and at higher modulation index.

The penalty due to PMD suffered by the direct detection CPFSK system at BER 10^{-9} is plotted in Fig. 5 as a function of DGD normalized by bit duration. It is observed from Fig. 5 that the penalty at BER = 10^{-9} is increasing with increasing values of the mean DGD. The amount of power penalty is found to be approximately 0.15 dB, 1.0 dB, 3.40 dB and 6.40 dB corresponding to mean DGD of 20 ps, 40 ps, 60 ps and 80 ps respectively. Further increase of DGD leads to a BER floor above 10^{-9} which can not be lowered by further increasing the signal power. It is observed from Fig. 5 and Fig. 6 that BER floors occur at about 10^{-4} and 10^{-8} for modulation index of 0.50 and 1.0 respectively to a corresponding mean DGD of 90 ps.

The plots of penalty due to other modulation schemes as reported in [8] are also shown in Fig. 7 for comparison. It is further noticed that penalty suffered by direct detection CPFSK with modulation index 0.50 and 1.0 is higher than that of NRZ-OOK system, but lower than RZ-OOK and NRZ-OOK system. For example, the penalty is 1.30 dB and 2.70 dB corresponding to DGD/bit duration ratio $(^{\langle\Delta\tau\rangle}/_{T})$ are 0.40 and 0.50 respectively for direct detection CPFSK system at modulation index 1.0. The corresponding penalty values are 2.30 dB, 1.70 dB and 2.90 dB, 3.80 dB for NRZ-OOK and RZ-OOK respectively.

5 Conclusion

An analytical approach is presented to evaluate the impact of polarization mode dispersion on the conditional and unconditional BER performance of a direct detection CPFSK system. Considering the Maxwellian distribution of the DGD, the results show that the power penalty due to PMD suffered by the direct detection CPFSK system is 4.0 dB corresponding to a mean DGD of 60 ps at modulation index of 1.0. Thus the amount of power penalty is higher at higher values of modulation index and higher values of the differential group delay between the two polarization axes. Also a direct-detection CPFSK system with modulation index of 0.50 suffers slightly higher amount of power penalty than a direct detection NRZ-DPSK system

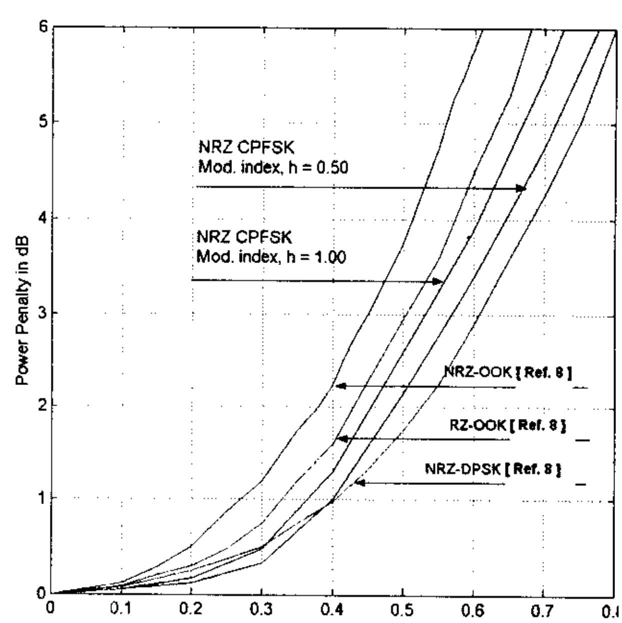

Fig. 7: PMD-induced penalty (for average BER) as a function of DGD/bit duration ($<\Delta\tau>/T$) for NRZ- and RZ-OOK, NRZ-DPSK and direct detection NRZ-CPFSK system

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